Dynamic response of reinforced concrete flexural members subjected to blast loading

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Blast loads have been a design concern for structural engineers for many years. Vapor Cloud Explosions (VCE) in petroleum refineries & petrochemical facilities, a bomb explosion within or immediately nearby a building results in extreme loading conditions and finally catastrophic damage on the building structure. Although such events may be relatively rare, when they do occur the consequences can be extremely severe involving personal casuality , financial loss and potentially impacting public safety. The analysis and design of structures subjected to blast loads requires a detailed understanding of blast phenomena and the dynamic response of various structural elements. In Blast resistant design, it is common practice to separate a structure into its major components for purposes of simplified dynamic analyses, though this do not consider dynamic interaction effects between the structural components. The paper explains the procedure to obtain the dynamic response of reinforced concrete flexural members subjected to blast loading. Finally a solved example is presented for a better understanding of the application of procedure.

EXPLOSIONS AND BLAST PHENOMENON

An explosion is defined as a large-scale, rapid and sudden release of energy. Explosions can be categorized on the basis of their nature as physical, nuclear or chemical events. Explosive materials can be classified according to their physical state as solids, liquids or gases.

The explosion generates hot gases under very high pressure and temperature. The hot gas expands forcing out the volume it occupies. As a consequence, a layer of compressed air (blast wave) forms in front of this gas volume containing most of the energy released by the explosion. Blast wave instantaneously increases to a value of pressure above the ambient atmospheric pressure. This is referred to as the side-on overpressure that decays as the shock wave expands outward from the explosion source. After a short time, the pressure behind the front may drop below the ambient pressure (Figure 1). During such a negative phase, a partial vacuum is created and air is sucked in. This is also accompanied by high suction

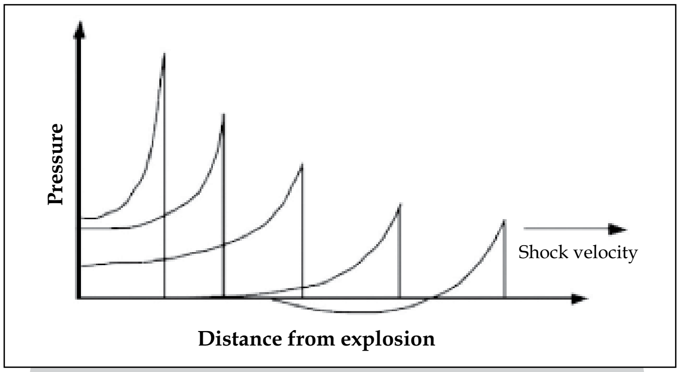


Figure 1. Blast wave propagation

winds that carry the debris for long distances away from the explosion source.

BLAST LOADING

The threat for a conventional bomb is defined by two equally important elements, the bomb size, or charge weight *W*, and the standoff distance *R* between the blast source and the target (Figure 2).

In the case of petroleum refineries & petrochemical facilities , the design blast loads are usually supplied by the facility owner based on site specific study conducted by owner's process safety specialist. The blast loads varies widely depending on the processes used in the facility.

The observed characteristics of air blast waves are found to be affected by the physical properties of the explosion source. Figure 2 shows a typical blast pressure profile. At the arrival time t_A, following the explosion, pressure at that position suddenly increases to a peak value of overpressure, P_{so} , over the ambient pressure, P_o. The pressure then decays to ambient level at time t_d, then decays further to an under pressure P_{so}^- (creating a partial vacuum) before eventually returning to ambient conditions at time $t_d+t_d^-$. The negative phase is of a longer duration and a lower intensity than the positive phase. The quantity P_{so} is usually referred to as the peak side-on overpressure, incident peak overpressure or merely peak overpressure [1] (TM 5-1300, 1990).

The incident peak over pressures P_{so} are amplified by a reflection factor as the shock wave encounters a structure in its path. Explosions extremely close to a structure impose a highly impulsive, high intensity pressure load over a localized region of the structure; while those farther away produce a lower-intensity, longer-duration uniform pressure distribution over the entire structure.

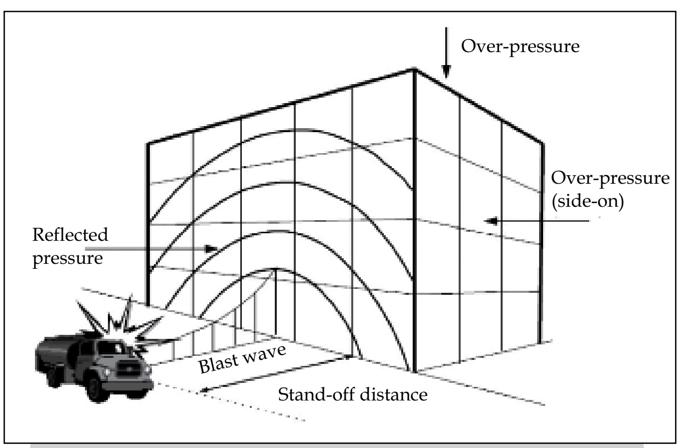


Figure 2. Blast loads on a building

A number of documents are available for computation of blast pressures such as (a) SG-22 [2] (b) Design of structures to resist nuclear weapon effects (ASCE Manual 42)(c)TM 5-1300 (d) Design of blast resistant buildings in petrochemical facilities - ASCE Publication [3] (e) OISD-STD-163 [4] and (f) IS: 4991-1968 [5]

For conventional high explosions, the peak over pressures depends upon charge weight *W* as an equivalent mass of TNT and the standoff distance *R* between the blast source and the structure. Some representative values of peak reflected overpressures with different *W-R* combinations is given in Table 1.

In the case of oil refineries and petrochemical facilities, the design blast pressures are usually supplied by the facility owner based on site specific study conducted by owner's process safety specialist which varies widely depending on the processes. The document SG-22 specifies that the structures spaced 30 meters from a vapor cloud explosion(VCE) hazard shall be designed for at least two blast over pressures as follows :

- High pressure, short duration, triangular shock loading : Side-on overpressure of 69 KPa with a duration of 20 milliseconds.

Table 1. Peak reflected over pressures(MPa)

W / R	100 kg TNT	500 kg TNT	1000 kg TNT	2000 kg TNT
1m	165.8	354.5	464.5	602.9
2.5m	34.2	89.4	130.8	188.4
5m	6.65	24.8	39.5	60.19
10m	0.85	4.25	8.15	14.7
15m	0.27	1.25	2.53	5.01
20m	0.14	0.54	1.06	2.13
25m	0.09	0.29	0.55	1.08
30m	0.06	0.19	0.33	0.63

- Low pressure, long duration, triangular loading : Side-on overpressure of 21 KPa with a duration of 100 milliseconds.

STRUCTURAL RESPONSE TO BLAST LOADING

Complexity in analyzing the dynamic response of blast-loaded structures involves the effect of high strain rates, the non-linear inelastic material behavior, the uncertainties of blast load calculations and the time-dependent deformations. Therefore, to simplify the analysis, a number of assumptions related to the response of structures and the loads have been proposed and widely accepted.

It is common practice to separate a structure into its major components with each component being represented by an equivalent elasto-plastic single degree of freedom (SDOF) system for purposes of simplified dynamic analyses. This uncoupled member by member approach do not consider dynamic interaction effects between major structural components. A series of separate SDOF dynamic analyses are performed for each of the primary structural components using the reaction time history of the supported member as loading input to the supporting member [3].

COMPUTATION OF DYNAMIC RESPONSE

The various steps in the computation of dynamic response for individual members are

Step1: load determination

Step2: Determination of member properties

Step3: Model representation

Step4: Trial member selection

Step5: Dynamic analysis

Step6: Deformation criteria check

A brief description of various steps are furnished below.

Step-1: Load determination

For computation of blast loads , the blast pressure – time history shown in Figure 3 is normally idealized to triangular shape and the negative phase is neglected. Although the actual blast load on an exposed element will vary over its tributary area, for design the maximum dynamic load (F_o) is typically taken as the product of this area and either the maximum pressure or a spatially averaged value.

If a rectangular structure is exposed to an explosion, it will be exposed to pressures on all its surfaces. Typical variations of blast load on different components are shown in Figure 4 [3].

Step-2 : Determination of member properties

The required dynamic properties usually include unit weight, modulus of elasticity and elastic yield strength. TM 5-1300 indicates that Grade 60 reinforcing bars have sufficient ductility for dynamic loading . Further a concrete of minimum compressive strength 27.6 Mpa shall be used to reduce the probability of shear failures [3,1].

In practice , the average yield strength of steel materials being installed is approximately 25 % greater than the specified min. values. *A strength increase factor(SIF)* is used to account for this condition. TM 5-1300 suggests using a 1.1 SIF for Grade 60 reinforcing. An *SIF* of 1.0 is considered for concrete.

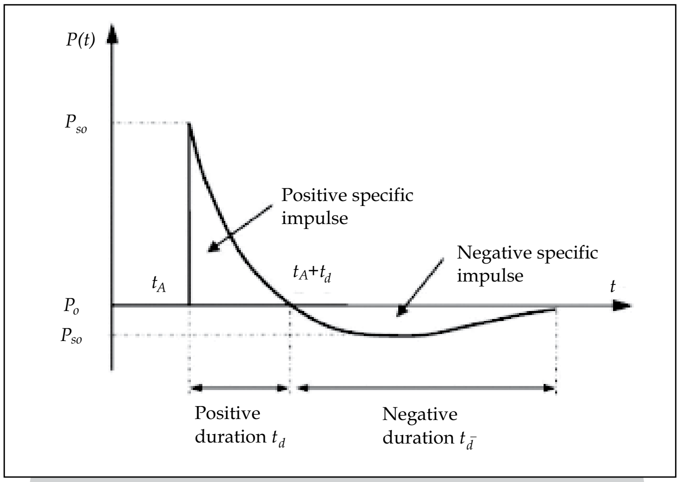


Figure 3. Blast wave pressure - Time history

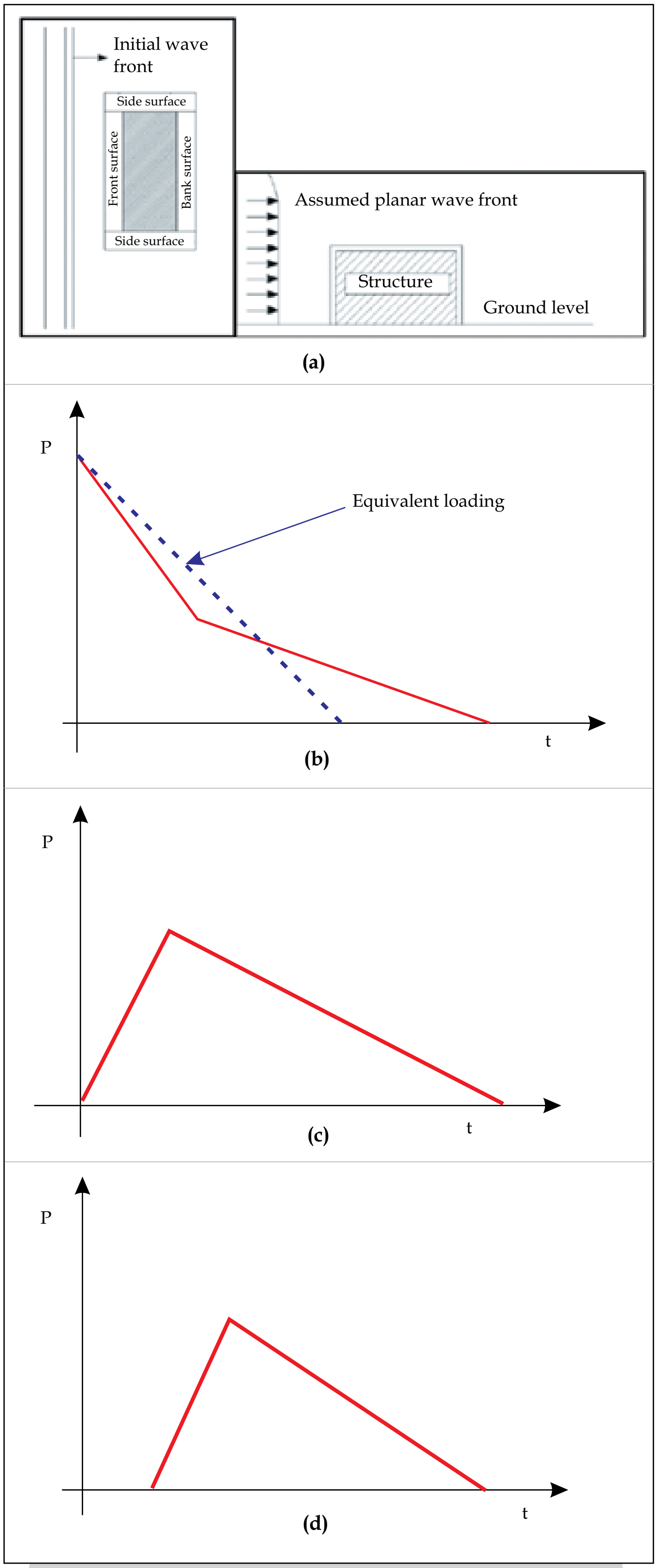


Figure 4. (a) Building subjected to blast wave (b) Front wall loading (c) Roof and side wall loading (d) Rear wall Loading

In practice Concrete and steel experience an increase in strength under rapidly applied loads. To incorporate the effect of material strength increase with strain rate , a *dynamic increase factor (DIF)* is applied to static strength values. Table 2 provides the *DIF* values for reinforcing bars, concrete & masonry for various stress types.

The dynamic strength for concrete and reinforcing steel are calculated as

$$f_{dc} = (SIF)(DIF)f_c$$

$$f_{dy} = (SIF)(DIF)f_y$$

where f_c and f_y are the characteristic strength of concrete and yield strength of reinforcing steel respectively.

Step-3: Model representation

The engineer must develop mathematical models for individual structural members. Individual members are usually idealized as simple one way beams or two way plates since these types of members can be adequately analyzed as equivalent SDOF systems with minimal engineering effort. One way members are the most common. Boundary conditions need to be assessed based on the type of connections to be used for the member supports.

Step-4: Trial member selection

Unlike most static design procedures, dynamic design requires a trial and error approach. For dynamic analysis, the required *resistance functions* for the member are determined from a trial section. The dynamic analysis

Table 2. Dynamic increase factors (DIF) (As per ASCE publication)[3]

Material and property	Failure mode	DIF
Concrete compressive strength	Flexure Compression Direct Shear	1.19 1.12 1.10
Masonry compressive strength	Flexure Compression Direct Shear	1.19 1.12 1.10
Deformed reinforcement steel yield strength	Flexure Compression Direct Shear Bond	1.17 1.10 1.10 1.17

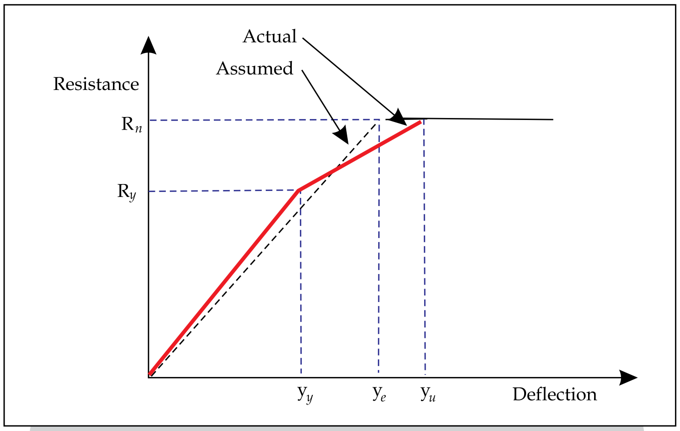


Figure 5. Resistance function for one-way elements

results indicate the adequacy of the trial section and the experience on the part of designer will help in reducing the number of iterations.

The simplified *resistance function* of a fixed/fixed or fixed/pinned element is trilinear as shown in Figure 5. Deflection consistent with the elastic stiffness k_1 occurs until initial plastic hinge formation at the yield capacity R_y, then the elastic-plastic stiffness k_2 governs up to the ultimate capacity R_n. Although it is possible to analyze the element using this resistance function, a common simplification that sacrifices little accuracy is to use an equivalent bi-linear stiffness calibrated to provide the same area under the curve and thus the same energy dissipation.

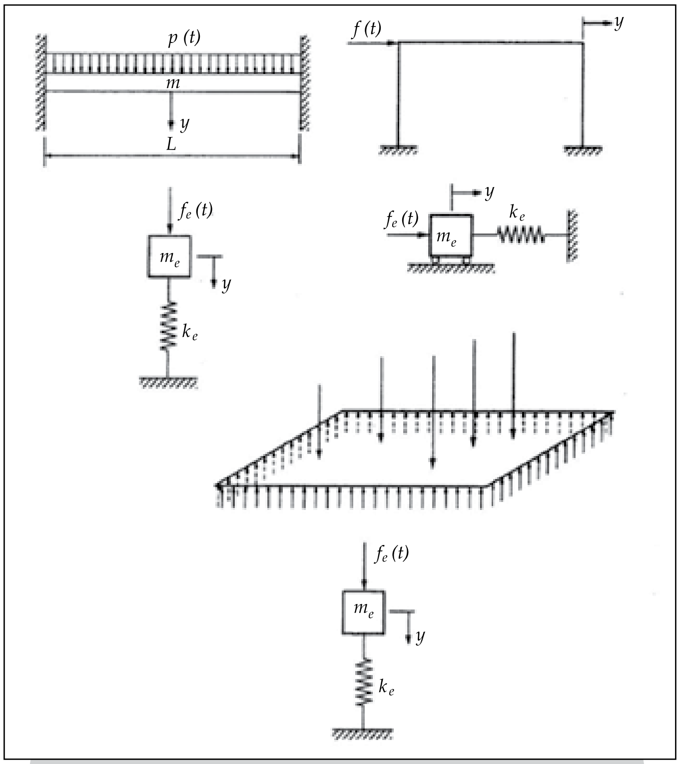


Figure 6. Equivalent single degree of freedom(SDOF)systems [6]

Step-5: Dynamic analysis

The purpose of this step is to compute member deformations and reactions. The analysis method shall provide the necessary balance between sufficient accuracy and calculation simplicity.

The basic analytical model used in most blast design applications is the elasto-plastic single degree of freedom (SDOF) system (Figure 6). It is common practice to separate a structure into its major components for purposes of simplified dynamic analyses though this do not consider dynamic interaction effects between the components. A series of separate SDOF dynamic analyses are performed for each of the primary structural components using the reaction time history of the supported member as loading input to the supporting member .

This member by member approach is illustrated in Figure 7 for a box type reinforced concrete shear wall building subjected to blast [3]. Front walls facing the blast are typically designed as a unit width one-way

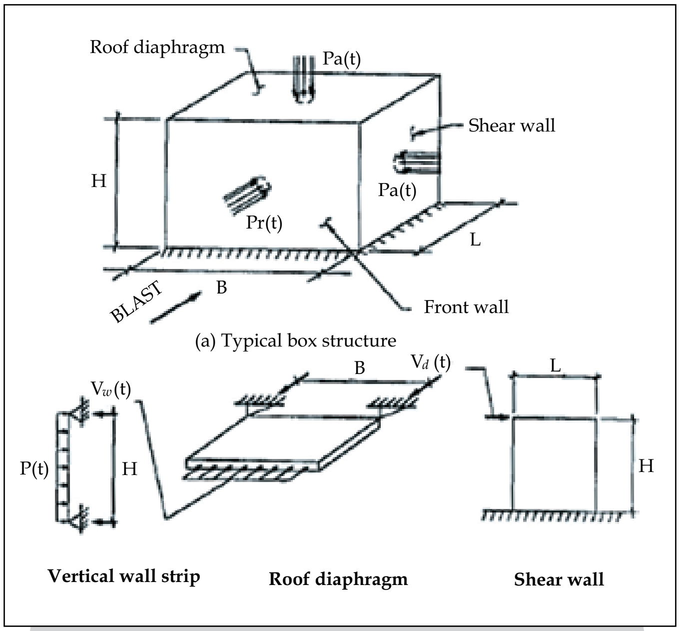


Figure 7. Forces acting on primary structural elements

member spanning vertically. Reaction time histories of a representative wall strip are used as loading input to the horizontal roof diaphragm which is supported by side walls oriented parallel to the direction of blast. The analysis proceeds from the front wall to the roof diaphragm to the side walls and finally to the foundation. A consistent load-path is thus established.

Having developed the bi-linear resistance curve for the member , an equivalent SDOF model shall be developed for the component in which the actual mass *(M)*, stiffness*(K)* and force *(F)* are to be converted into an equivalent mass, stiffness and force based on the support conditions, member loading and stress level(elastic/ plastic). The procedure for obtaining an equivalent SDOF approximation for a structural component is based on its deformed shape under the applied loading and the strain energy equivalence between the actual structure and the SDOF approximation.

Equivalent mass, stiffness and loading are obtained through the use of *transformation factors* [6] (Biggs-1964). Transformation factors are used to obtain appropriate properties for the equivalent SDOF system as follows:

Equivalent stiffness $K_e = K_L * K$

Equivalent mass $M_e = K_M * M$

Equivalent Force $F_e = K_L * F$

Equivalent Resistance $R_e = K_L * R$

Where K_L = load or stiffness transformation factor , given by

$$K_L = \frac{\int_0^L p(t)\emptyset(x)dx}{p(t)L}$$

K_M = mass transformation factor given by

$$K_M = \frac{\int_0^L m(x)\,[\emptyset(x)]^2 dx}{M}$$

In the above equations $\emptyset(x)$ is the assumed shape function representing the deflected shape of the actual structure and the magnitude of these functions is set by the requirement that $\emptyset(L/2)=1$

The transformation factors for one-way members for simply supported boundary conditions are given below in Table 3. For other boundary conditions reference can be made to Biggs-1964 [6].

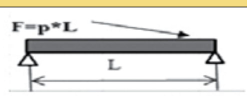
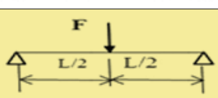
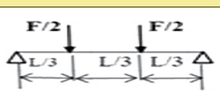
Due to the short time in which the structures reaches its maximum response , damping have little effect on peak displacements and is conservatively ignored in blast resistant design. Hence the dynamic equilibrium equation for an equivalent SDOF can be written as

$K_{LM} Ma + Ky = F(t)$; Where $K_{LM} = K_M / K_L$

M = mass, *a* = acceleration, *K* = stiffness, *y* = displacement, *F(t)* = applied blast load as a function of time .

After obtaining an equivalent SDOF model, the response of the same shall be obtained. If the structural designer is

Table 3. Transformation factors for one way members (As per Biggs) [6]

Loading diagram	Strain range	Load factor K_L	Lumped mass factor K_M (1)	Uniform mass factor K_M	Bending resistance (R_b)	Spring constant (K)	Dynamic reaction (V)
	Elastic Plastic	0.64 0.50	----- -----	0.50 0.33	$8M_{pc}/L$ $8M_{pc}/L$	$384EI/5L^3$ 0	0.39R+0.11F $0.38R_u$+0.12F
	Elastic Plastic	1.00 1.00	1.00 1.00	0.49 0.33	$4M_{pc}/L$ $4M_{pc}/L$	$48EI/L^3$ 0	0.78R-0.28F $0.75R_u$-0.25F
	Elastic Plastic	0.87 1.00	0.76 1.00	0.52 0.56	$6M_{pc}/L$ $6M_{pc}/L$	$56.4EI/L^3$ 0	0.525R-0.025F $0.52R_u$-0.02F

Note (1) : Equal portions of the concentrated mass are lumped at each concentrated load
(2) : M_{pc} is the ultimate moment capacity at midspan

interested only in maximum displacement, the response charts already available for different load-time functions can be used. One such chart for triangular load is shown in Figure 8 for reference. For other load-time functions reference can be made to Biggs-1964[6].

On the other hand if the engineer requires complete response of the system, it would be better to use various available numerical analysis techniques to perform the time history analysis. Newmark's numerical integration procedure (constant/linear acceleration) is most commonly used to obtain the time history response for nonlinear SDOF systems. To obtain an accurate and numerically stable solution, time increment of 1/10 of either load duration or natural period of member whichever is smaller is selected.

The author has developed a computer program in MATHCAD using Newmark's method (Linear acceleration)to perform the time history analysis of blast loaded members with bi-linear resistance curve [7]. The explanations of various symbols are available at Chopra, A.K [8]

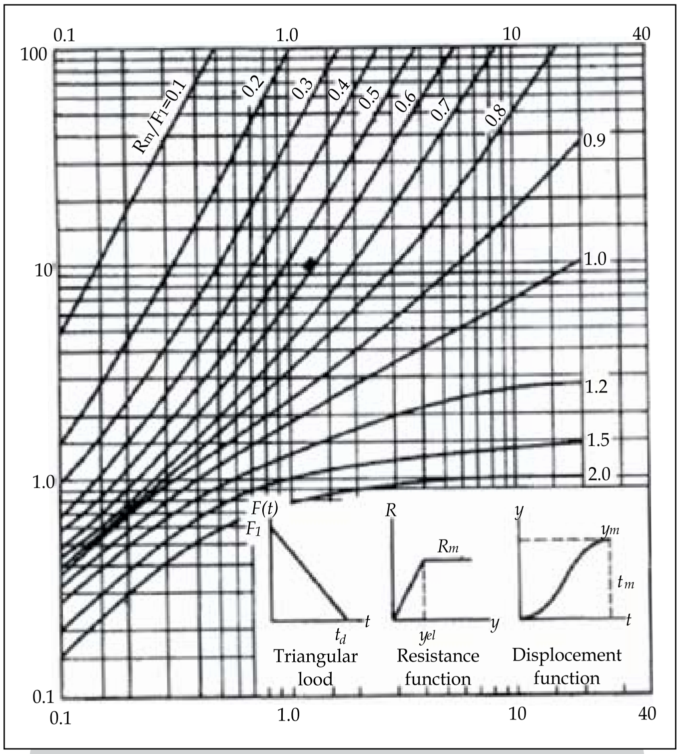


Figure 8. Maximum response of elsto-plastic SDOF system (undamped) due to triangular load pulse with zero rise time [6]

Step-6: Deformation criteria check

In the case of blast loaded members, as the stresses reach or exceed yield stresses, the adequacy of assumed member section shall be verified after the dynamic analysis on the basis of maximum member deformations. Different documents use various acceptance criteria such as ductility ratio and hinge rotations. *Ductility ratio* is defined as the maximum displacement of the member divided by the displacement at the elastic limit and is commonly designated by the symbol μ. It is a measure of the degree of inelastic response experienced by the member. *Hinge rotation* is another measure of member response which relates maximum deflection to span and indicates the degree of instability present in critical areas of the member. It is designated by symbol θ.

The deformation criteria depends on the assumed level of protection (Low, Medium or High) used in the design. For example the ASCE publication [3] uses support rotation limits 2,4 & 8 degrees for slab members corresponding to low, medium and high levels of protection.

COMPUTER PROGRAMS FOR BLAST AND SHOCK EFFECTS

Computational programs for blast prediction and structural response can be categorized into uncoupled and coupled analyses. The uncoupled analysis calculates blast loads as if the structure (and its components) were rigid and then applying these loads to a responding model of the structure. For a coupled analysis, the blast simulation module is linked with the structural response module and thus model for blast-load prediction is solved simultaneously with model for structural response. By accounting for the motion of the structure while the blast calculation proceeds, the pressures that arise due to motion and failure of the structure can be predicted more accurately. Examples of this type of computer codes are AUTODYN, DYNA3D, LSDYNA and ABAQUS.

CONCLUDING REMARKS

The paper makes an attempt to detail the procedure & concepts to obtain the dynamic response of blast loaded reinforced concrete flexural members. Though this approach is widely used in industries and engineering companies, the design engineer should be aware of the inherent limitations of the method such as it does not consider the dynamic interaction between connected members. In cases where such interactions are critical, the use of finite element software shall be resorted to. Further the accuracy of the SDOF approach depends on the mass modeling , use of proper resistance function and computation of member stiffness based on stress levels.

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APPENDIX

Example

A reinforced concrete shear wall box structure shown below in Figure A1 is subjected to a blast load. Compute the dynamic response of the front wall.

Solution :

The problem is solved in MATHCAD. The program is split in two parts. One part computes the parameters of equivalent SDOF for the component. These parameters are used as input to second part which calculates the dynamic response of the system including displacement, velocity, acceleration, resistance and dynamic reaction. ACI-318 provisions are used for member resistance function calculations [9]

Part-1 : Computation of equivalent SDOF system parameters

MPa = 1 N/mm^2 kN = 1000 N

Yield strength of reinforcing steel f_y = 414 MPa
Characteristic compressive strength of concrete f_c = 27.6 MPa

Modulus of elasticity of concrete E_c = 24856 MPa
Modulus of elasticity of steel E_s = 199948 MPa
Unit weight of concrete γ_c = 23.6 kN/m^3
Strength Increase Factor for concrete SIF_c = 1.0
Strength Increase Factor for reinforcing steel SIF_y = 1.1
Dynamic Increase Factor for concrete DIF_c = 1.19
Dynamic Increase Factor for reinforcing steel DIF_y = 1.17
Strength Increase Factor for concrete in shear SIF_v = 1.0
Dynamic Increase Factor for concrete in shear DIF_v = 1.0

Acceleration due to gravity g = 9.807 m/s^2
Response limit θ_{lim} = 2 degrees

As the length to height ratios of the walls are higher than 2.0, they will be analyzed as one-way beams pined at the top slab & floor slab level (proper detailing shall be considered to ensure this assumption). Vertical steel will be provided on both faces

Wall width considered for analysis b = 0.305 m
Wall thickness t = 254 mm
Span L = 3.66 m

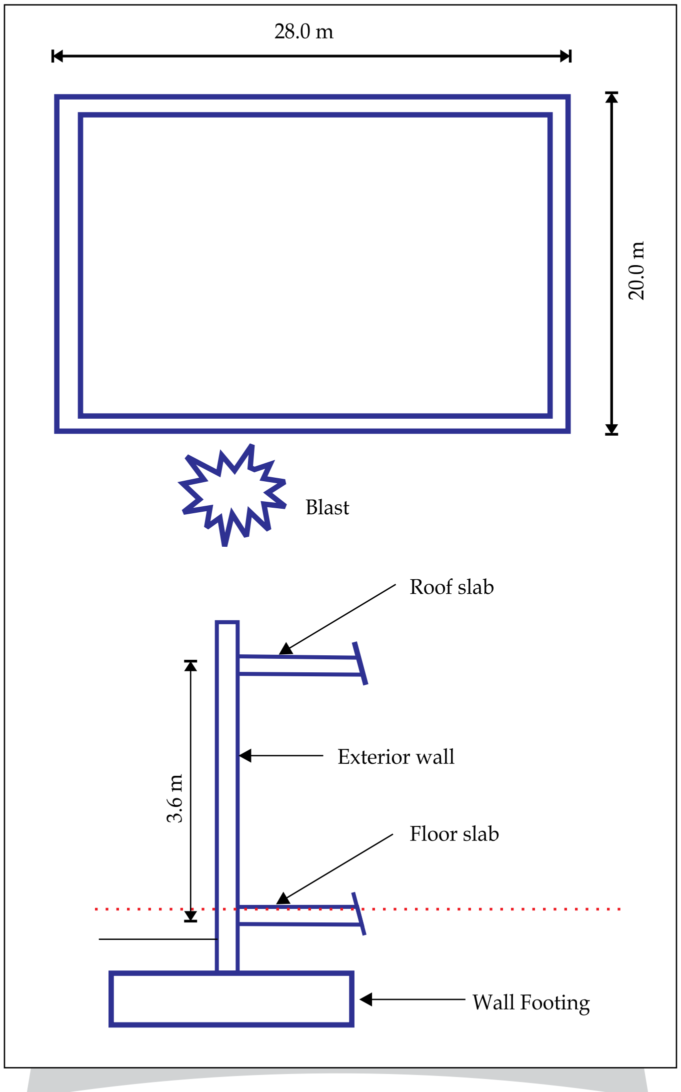
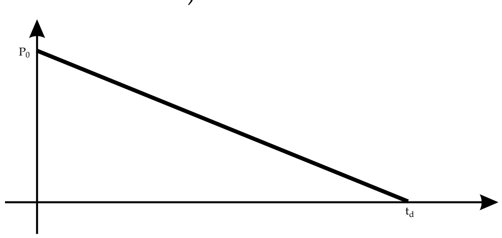


Figure A1. (a) Plan (b) Elevation

Bar dia ϕ = 16 mm
Clear cover cc = 38 mm
Steel provided(on one side)-value to be checked later A_{st} = 400 mm^2

Blast load parameters (Computation of pressure & loads not shown here)



Peak load P_0 = 44 kN
Duration time t_d = 0.05 sec

Computation of bending resistance

For dynamic bending , the design stresses for concrete and steel are given by

$f_{dc} = (SIF_c)(DIF_c)\ f_c$ $\quad$ f_{dc} = 32.844 MPa
$f_{dy} = (SIF_y)(DIF_y)\ f_y$ $\quad$ f_{dy} = 532.818 MPa
Effective depth $\quad$ d= t-cc- ϕ/2 $\quad$ d = 208 mm
Percentage of steel $\quad$ $\rho = A_{st}/(bd) = 6.305*10^{-3}$

Min. steel (ACI-318) $\rho_{min} = 1.40/f_{dy} = 2.628*10^{-3}$ (Hence ok.)
Depth of nuetral axis (ACI-318) $a = A_{st}*f_{dy}/(0.85f_{dc}*b)$ = 25.03 mm
Moment of resistance (plastic) $M_p = A_{st}*f_{dy}*(d-a/2)$ = 41.663 kNm
Bending resistance $R_b = 8M_p/L$ = 91.067 kN

Computation of shear resistance

For dynamic shear , the design stresses for concrete is given by
$f_v = (SIF_v)(DIF_v)\ f_c$ $\quad$ f_v = 27.6 MPa
Shear capacity (ACI-318) $V_n=0.17*\sqrt{fv}*b*d$ = 56.659 kN

The critical section for shear is at a distance "d" from support, therefore the shear resistance is
$R_s = V_n*L/(0.5*L-d)$ $\quad$ R_s = 127.849 kN

Computation of SDOF equivalent system

Ultimate resistance R_u = 91.067 kN (bending controls)
Gross moment of inertia of section $I_g = bt^3/12 = 4.165*10^4$ cm^4
Modular ratio $n = E_s/E_c$ = 8.044

Neutral axis depth for cracked section is given by

$$a_c = \frac{-n*Ast+\sqrt{n*Ast*(n*Ast+2bd)}}{b}$$ $\quad$ a_c =56.352 mm

Cracked moment of inertia of section

$$I_{cr} = \frac{b*a_c{}^3}{3} + n * A_{st} * (d - a_c)^2$$

$I_{cr} = 9.219*10^3$ cm^4

Average moment of inertia $I_a = (I_g+I_{cr})/2 = 2.543*10^4$ cm^4
Effective stiffness $K = 384E_cI_a/(5L^3) = 9.903*10^3$ kN/m
Yield deflection $y_e = R_u/K$ = 0.92 cm
Beam mass $\quad$ $M=btL\gamma_c/g$ = 0.682 $kN.sec^2/m$

Table A1. Numerical integration solution

time (sec)	Force (kN)	y (m)	v (m/sec)	a (m/sec2)	resistance (kN)	reaction (kN)
0	44	0	0	89.43	0	5.06
0.002	42.24	0.0001741	0.1718	82.35	1.725	5.522
0.004	40.48	0.0006733	0.3229	68.72	6.668	7.222
0.006	38.72	0.001444	0.4412	49.64	14.3	9.957
0.008	36.96	0.00241	0.5175	26.61	23.87	13.44
0.01	35.2	0.003481	0.5455	1.469	34.48	17.32
0.012	33.44	0.004559	0.5232	-23.79	45.14	21.23
0.014	31.68	0.005542	0.4523	-47.16	54.88	24.77
0.016	29.92	0.006339	0.3383	-66.78	62.78	27.61
0.018	28.16	0.006873	0.1905	-81.1	68.06	29.44
0.02	26.4	0.007086	0.02039	-88.97	70.17	30.05
0.022	24.64	0.006948	-0.1584	-89.78	68.81	29.33
0.024	22.88	0.006456	-0.3316	-83.45	63.94	27.25
0.026	21.12	0.005635	-0.4855	-70.49	55.8	23.91
0.028	19.36	0.004535	-0.608	-51.94	44.91	19.52
0.03	17.6	0.003231	-0.6892	-29.25	31.99	14.34
0.032	15.84	-0.00181	0.7227	-4.245	17.93	8.724
0.034	14.08	0.0003735	-0.7058	21.1	3.699	3.043
0.036	-12.32	0.0009801	-0.6399	44.77	-9.706	-2.32
0.038	-10.56	0.002157	-0.5303	64.88	-21.36	-7.01
0.04	-8.8	0.003078	-0.3856	79.84	-30.48	-10.72
0.042	-7.04	0.003684	-0.2173	88.45	-36.48	-13.23
0.044	-5.28	0.00394-0	0.03879	90.04	-39.02	-14.42
0.046	-3.52	0.003841	0.1357	84.47	-38.04	-14.24
0.048	-1.76	0.003409	0.2924	72.2	-33.76	-12.8
0.05	0	0.002692	0.4188	54.18	-26.66	-10.26
0.052	0	0.001759	0.5084	35.4	-17.42	-6.705
0.054	0	0.0006855	0.5576	13.8	-6.788	-2.613
0.056	0	0.0004421	0.5624	-8.899	4.378	1.686
0.058	0	0.001535	0.5227	-30.89	15.2	5.851
0.06	0	0.002505	0.4414	-50.42	24.81	9.551
0.062	0	0.003277	0.325	-65.95	32.45	12.49
0.064	0	0.003788	0.1828	-76.24	37.51	14.44
0.066	0	0.003998	0.02607	-80.47	39.59	15.24
0.068	0	0.003891	-0.1327	-78.31	38.53	14.83
0.07	0	0.003474	-0.281	-69.93	34.41	13.25

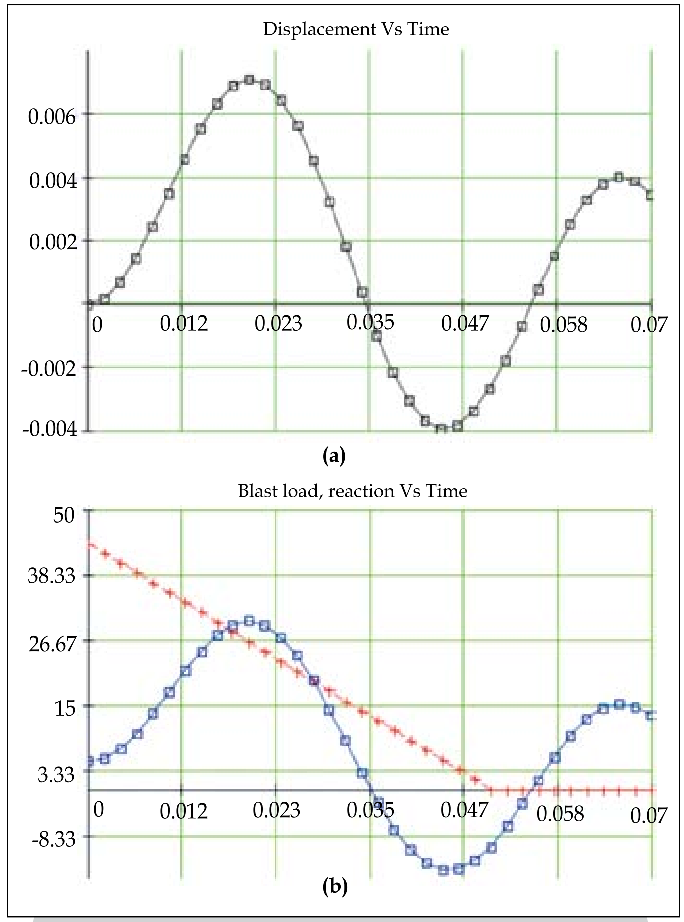


Figure A2. (a) Time versus displacement (meter) (b) Time versus blast load & dynamic reaction(kN)

Transformation factors for elastic and plastic conditions

$K_{Le} = 0.64$ $K_{Lp} = 0.50$ $K_{Me} = 0.50$ $K_{Mp} = 0.33$

$K_{LMe} = K_{Me}/K_{Le} = 0.781$ $K_{LMp} = K_{Mp}/K_{Lp} = 0.66$

Because of the expected response an average value of K_{LM} will be used .

$K_{LM} = (K_{LMe} + K_{LMp})/2 = 0.721$

Equivalent mass $M_e = K_{LM}.M = 0.492$ kN.sec^2/m

Period of vibration $t_n = 2\pi\sqrt{\frac{M_e}{K}} = 0.044$.sec

Time increment to be used $\Delta t_a = t_n/10 = 4.427*10^{-3}$ sec. However considering the fact that the wall is supported to much stiffer elements use a much smaller value of time increment

Use $\Delta t = 0.002$.sec

Part-2 : Computation of dynamic response of equivalent SDOF system

Dynamic response of the SDOF system is evaluated using the Newmark's linear acceleration method implemented in MATHCAD and the results are presented below. The response quantities are given in Table A1. The displacement response of the system is shown in Figure A2(a) and the variation of Dynamic reaction and applied load with time is shown in Figure A2(b).

The positive peak deflection is y_m= 7.10 mm at t=0.02 sec. (refer Table A1)

The support rotation is given by $\tan\theta_d$=(ym/0.5L) =(7.10/0.5* 3660)

θ_d= 0.22 degrees < 2 degrees . Hence the trial section is O.K