Optimisation of concrete multicellular bridge decks

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The paper presents an optimization study of prismatic multicellular bridge decks for minimum cost. The bridge decks have been analysed for dead load; IRC class AA tracked vehicle loading, class AA wheeled vehicle loading and class A train of vehicles loading. The analyses have been carried out by finite strip method. A parametric study has been conducted to determine the influence of various parameters on the cost of cellular decks per metre run. The parameters considered are the number of cells and span length. A cross-sectional configuration consisting of three cells has been analysed for each of the 20m, 25m, 30m, 35m, and 40m spans. The span giving the minimum cost per metre run has been analysed for different numbers of cells to obtain the economic configuration.

The cellular bridge decks are being increasingly used for spanning large lengths because they provide desirable aesthetic properties and high torsional stiffness. Due to high torsional stiffness they are structurally efficient giving a good strength-weight ratio, and their clean lines give them an attractive appearance. A cellular bridge deck consists essentially of a 3-D assemblage of plate elements, with each element having stiffness against both in-plane and flexural stresses. The primary structural actions are: longitudinal bending brought about by direct stresses in top and soffit slabs and in-plane shear in web; torsion caused by the in-plane shear stresses in top and soffit slabs and in the webs.

The conventional approach of designing the reinforced concrete bridges consists of using the piling-up procedure shown in Figure 1, which assumes the deck slab, the longitudinal girders and the cross beams or diaphragms acting independently. Although this simplification removes the complexity of analysis, it ignores the interaction between individual structural elements, thereby leading to an uneconomical design. In this approach, the effect of torsion is completely neglected.

On the other hand, the finite element method possesses the necessary versatility to cope with such complex structures, but the cost of using the technique for multicellular deck is, however, prohibitive. An alternate method of analysis using finite strip method is much more economical for predicting the

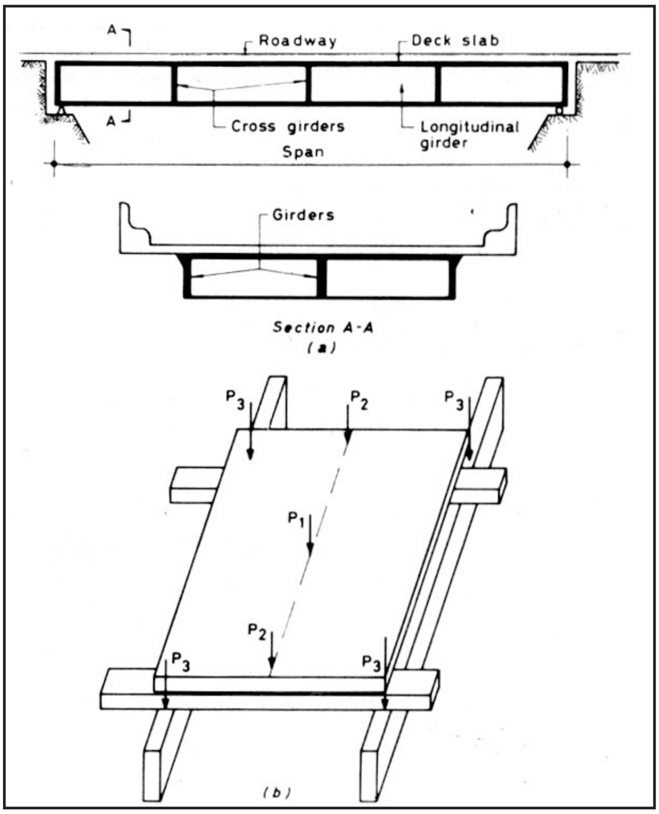

Figure 1. (a)Typical reinforced concrete bridge deck (b) piling up of elements

Notation

Subscripts and superscripts

c	for concentrated load
I	one edge of the strip
j	other edge of the strip
m	for mth harmonic
x,y	refer to the quantities related to x and ydirections
(•)	derivative with respect to ξ

Variables

a	=	width of the strip
{F} m	=	load vector corresponding to m^{th} harmonic
{K}mm	=	stiffness matrix of a strip for m^{th} harmonic with respect to local axes
I	=	span of the deck
u, v, w	=	displacement components of the strip along the co-ordinate axes
x	=	distance along X-axis
Y_m	=	series part of the displacement function for m^{th} harmonic, for simply supported deck, $y_m = \sin\frac{M\pi Y}{l}$
Y'_m	=	derivative of Y_m
θ	=	rotation about y-axis
ξ	=	dimensionless co-ordinate (ξ = x/a) varying from 0 to 1
$\{d\Delta\}_m$	=	displacement vector corresponding to m^{th} harmonic

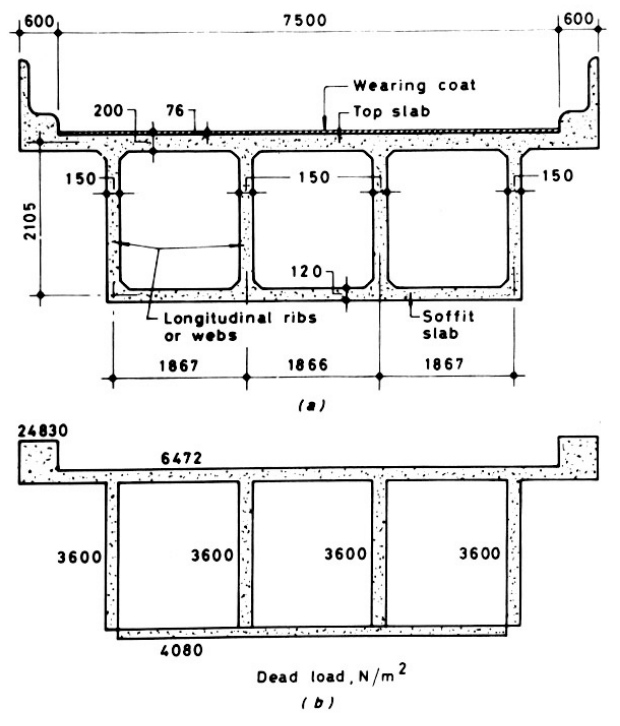

Figure 2. (a)Typical cross-section of a 3-cell cellular bridge deck (b) idealisation for dead load

design forces. This paper is devoted to an investigation of multicelluar bridge decks using the finite strip method. A 3-cell cellular bridge deck shown in Figure 2 has been analyzed for 20m to 40m spans. The analyses have been carried out for various IRC loadings. The optimum span giving the minimum cost has been analyzed with different numbers of cells. The configuration giving overall minimum cost /m of the bridge has been determined.

Modelling of the deck

The cellular bridge deck shown in Figure 2(a) is assumed to be composed of plane finite strips of the type shown in Figure 3(b) with four degrees of freedom at each nodal line. The degrees of freedom are in-plane displacements u and v. the vertical displacement w, and the rotation θ about yaxis. The displacements u, v and w are in the direction of the three orthogonal axes x, y and z, respectively. The force vector, (F m is related to the displacement vector $\{d\Delta\}_m$ as

$$[F]m = [k]mm\ [d\Delta]m \qquad \ldots(1)$$

$$\text{Where } [d\Delta]m = [u_i\ v_i\ w_i\ \theta_i\ u_j\ v_j\ w_j\ \theta_j]^T_m \qquad \ldots(2)$$

The matrix [k]mm represents the stiffness matrix for m^{th} harmonic component of displacement.

$$\left\{\frac{u}{v}\right\} \cdot \sum_{m=1}^{r} \left[\frac{(1-\delta)ym \quad \circ \quad \delta\ ym \quad \circ}{\circ\ \frac{(1-\delta)lym}{m\ e} \quad \circ\ \frac{\delta lym}{m\ e}}\right] \begin{bmatrix} uj \\ vj \\ uj \\ vj \end{bmatrix} \qquad \ldots(3)$$

$$\{w\} \cdot \sum_{m=1}^{r} [(1-3\varepsilon^2+2\varepsilon^3)\ x\ (1-2\varepsilon+\varepsilon^2)(3\varepsilon^2-2\varepsilon^3)]$$

$$x\ (\varepsilon^2-\varepsilon) \begin{bmatrix} wj \\ \theta j \\ wj \\ \theta j \end{bmatrix}_m y_m \qquad \ldots(4)$$

where, $Ym = \sin\frac{m\,\pi\,y}{l}$ and $Ym = \frac{m\,\pi}{l}\cos\frac{m\pi y}{l}$ and ξ = x/a which varies from zero to one.

The strip stiffness matrix and load vector given in equation (1) are in terms of local axes. In the case of cellular bridge decks, the prismatic strips generally meet at an angle. Therefore, in order to establish the equilibrium of the nodal forces, the quantities in terms of local axes are transferred to global axes. In case of simply supported strips the terms of the series are uncoupled and the stiffness matrix of each term can be found, assembled and solved separately. The resulting matrix equations are then solved for edge displacements for the m^{th} harmonic of given load. The stress components of various harmonics are superimposed to get complete solution. The analysis is based on the following assumptions:

1. The material of the bridge is homogeneous, isotropic or orthotropic.

2. The deflections are small as compared to the geometric dimension of the structure.

3. The in-plane forces cause only in- plane deformations and

forces normal to the plane of the element cause flexural deformations only.

4. The cellular bridge deck at the ends is supported by the diaphragms, which are infinitely stiff in their planes, but infinitely flexible out of planes. This is to achieve the idealized simply-supported conditions.

5. The IRC patch loads are considered as point or line loads, depending upon the size of the patch.

The load vector depends upon the type of load. For concentrated load, the vector is:

$$[F]_m = P_c\, y_m(yc) \begin{Bmatrix} (1-3\varepsilon^2+2\varepsilon^3) \\ x_c\,(1-2\varepsilon+\varepsilon^2) \\ (3\varepsilon^2-2\varepsilon^3) \\ x_c(\varepsilon^2-\varepsilon) \end{Bmatrix} \quad \ldots(5)$$

Where, P_c is the concentrated load at a point $x = x_c$ $y = yc$, ym(yc) is the value of the series function at y_c and $\xi = \frac{x_c}{a}$. Generally the structure is discretized such that the nodal line is placed below the point or line load. Therefore, x_c becomes equal to zero and the load vector of the point load reduces to:

$$[F]_m = P_c\, y_m(yc) \begin{Bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad \ldots(6)$$

where $y_m(y_c) = \sin\left(\frac{m\propto y_c}{l}\right)$ to the point load ...(7)

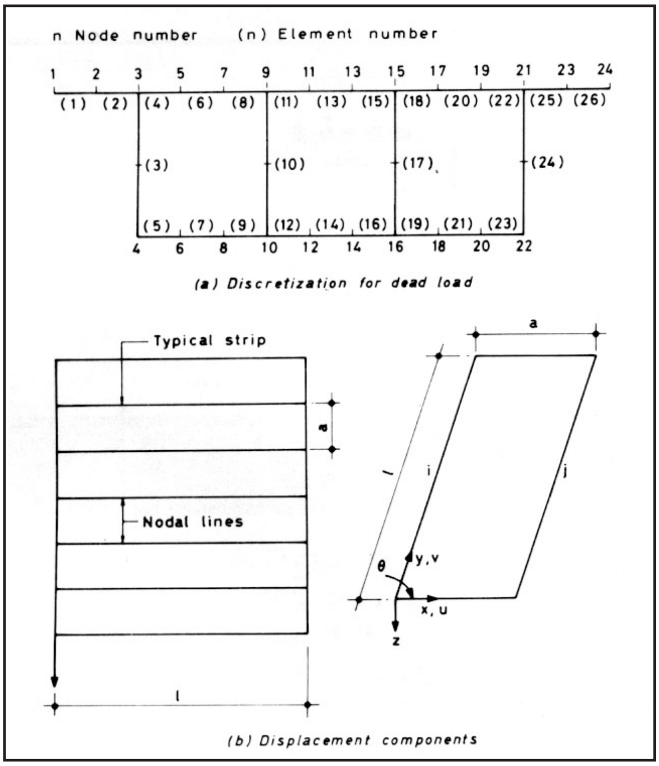

Figure 3. Discretisation scheme

For uniformly distributed load of intensity q, the load vector is given by:

$$F_m = \frac{ql}{l2} \begin{Bmatrix} 6 \\ 1 \\ 6 \\ -1 \end{Bmatrix} \int_o^l y_m\, d_y \quad \ldots(8)$$

For an in-plane concentrated load P_{uc}, the load vector tor a simply-supported strip is given by:

$$[F]_m = P_{uc} \begin{Bmatrix} (1-\varepsilon) \\ \varepsilon \\ (1-\varepsilon) \\ \varepsilon \end{Bmatrix} \sin\left(\frac{m\propto}{t}\, y_c\right) \quad \ldots(9)$$

where, the in-plane point load is acting at $x = x_c$ and $y = y_c$ and $\xi = \frac{x_c}{a}$

Similarly, for in-plane load P_{vc}.

$$[F]_m = \frac{ql}{l2} \begin{Bmatrix} 6 \\ 1 \\ 6 \\ -1 \end{Bmatrix} \int_o^l y_m\, d_y \quad \ldots(10)$$

In case the nodal line is placed at the point where the concentrated load acts $\xi = 0$. For a line load P of length c the force vector is:

$$[F]_m = \frac{4P}{nx} \sin\left(\frac{n\,x\,c}{t}\right) \sin\left(\frac{n\,x\,u}{t}\right) \begin{Bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad \ldots(11)$$

Based on the finite strip theory outlined above, a computer programme STRIP has been developed to analyse the prismatic structures. To handle the patch loads of IRC standard loadings, a subroutine SUM FOR has been incorporated to sum up the effect of various wheel loads. The programme has been implemented on DEC 2050 machine at the Regional Computer Centre, Chandigarh.

Parametric study

The parameters involved in the study of a multicelluar bridge deck are:

1. spans, I, of the bridge deck
2. number of cells in the cross-section.

The variation of these parameters is given in Table 1.

Table 1. Multicell bridge deck cases analysed

Sr. No.	Span Length, m	Number of cells
1	20	3
2	25*	2,3,4
3	30	3
4	35	3
5	40	3

* For the 3-cell configuration, the 25-m span was the optimum and analysed with different numbers of cells.

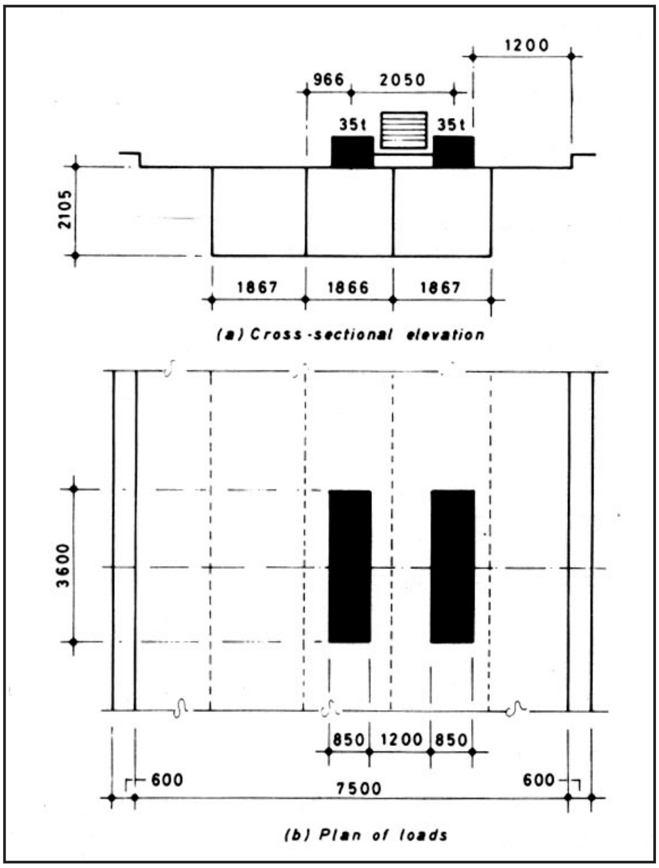

Figure 4. Placement of class AA tracked loading on the deck

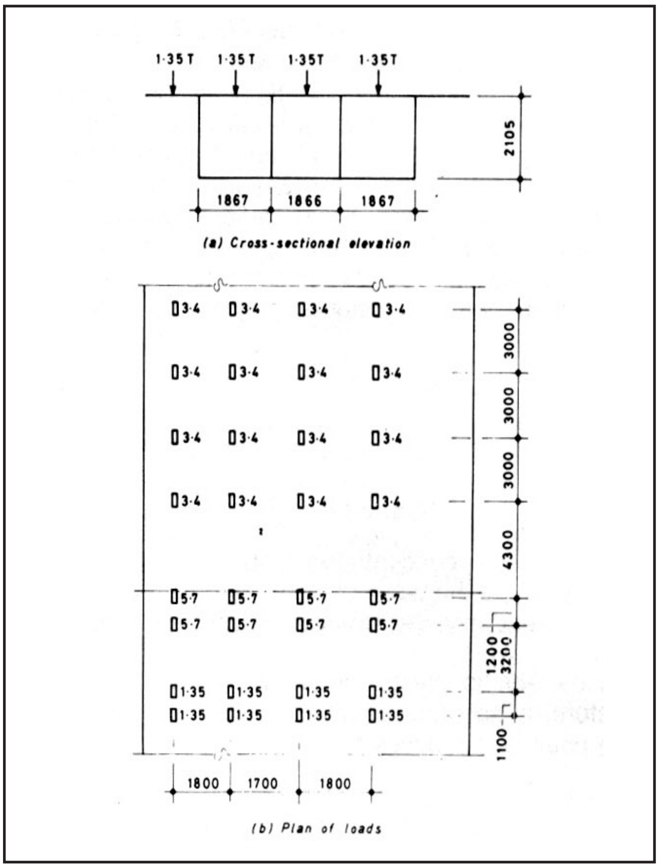

Figure 5. Placement of class AA wheeled loading on the deck

A discretization scheme with 26 finite strips as shown in Figure 3(a) has been used for the analysis of 3-cell cellular bridge decks. The nodal lines have been numbered in such a way that the absolute maximum difference between any two adjacent nodal lines is minimum. Fifteen harmonics have been found to give sufficiently accurate results, within a 4-percent range.

The cellular bridge decks have been analysed for dead load and live load including impact, separately. The dead load due to kerb is also taken into account. The positioning of various IRC loadings for maximum stresses has been shown in Figures 4 to 6. For IRC class AA and class A loadings, the loads are placed such that there is maximum eccentricity between the centre of gravity of the cross-section and centre of gravity of the loads. The critical combination of dead and live loads giving maximum design stresses has been obtained. The design stresses obtained are transverse moment, longitudinal stresses and torsional stresses. For a given configuration, the thicknesses of the top and soffit slabs, and longitudinal ribs or webs are initially assumed and their adequacy tested after analysis and modified, if necessary. The analysis and design is repeated till the design is found to be safe. The minimum thickness of the top slab is kept as 200 mm and that of ribs as 150mm due to practical considerations. M20 concrete and high yield strength deformed bars have been used in the design. The variation of various stresses is shown in Figures 7 and 8. The reinforcement details for a typical bridge deck are shown in

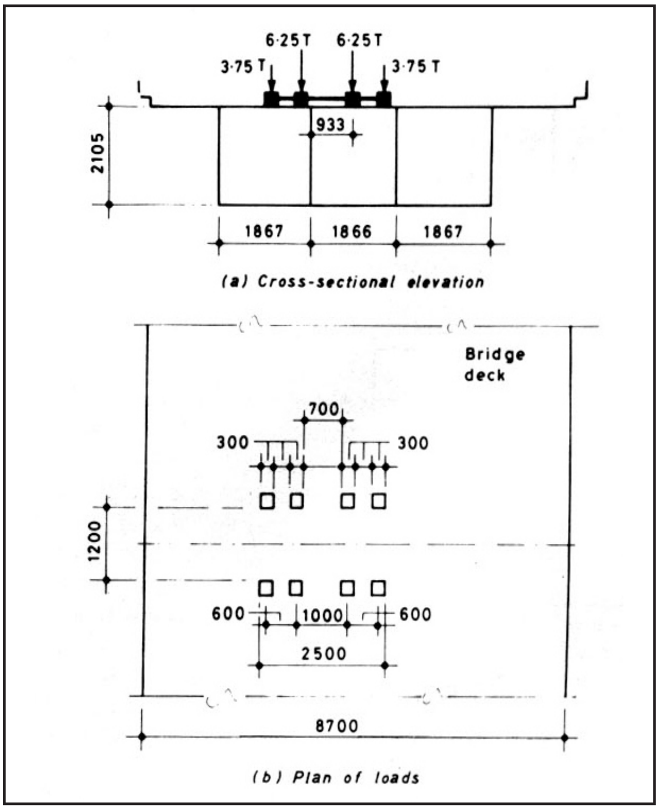

Figure 6. Placement of class A loading on the deck

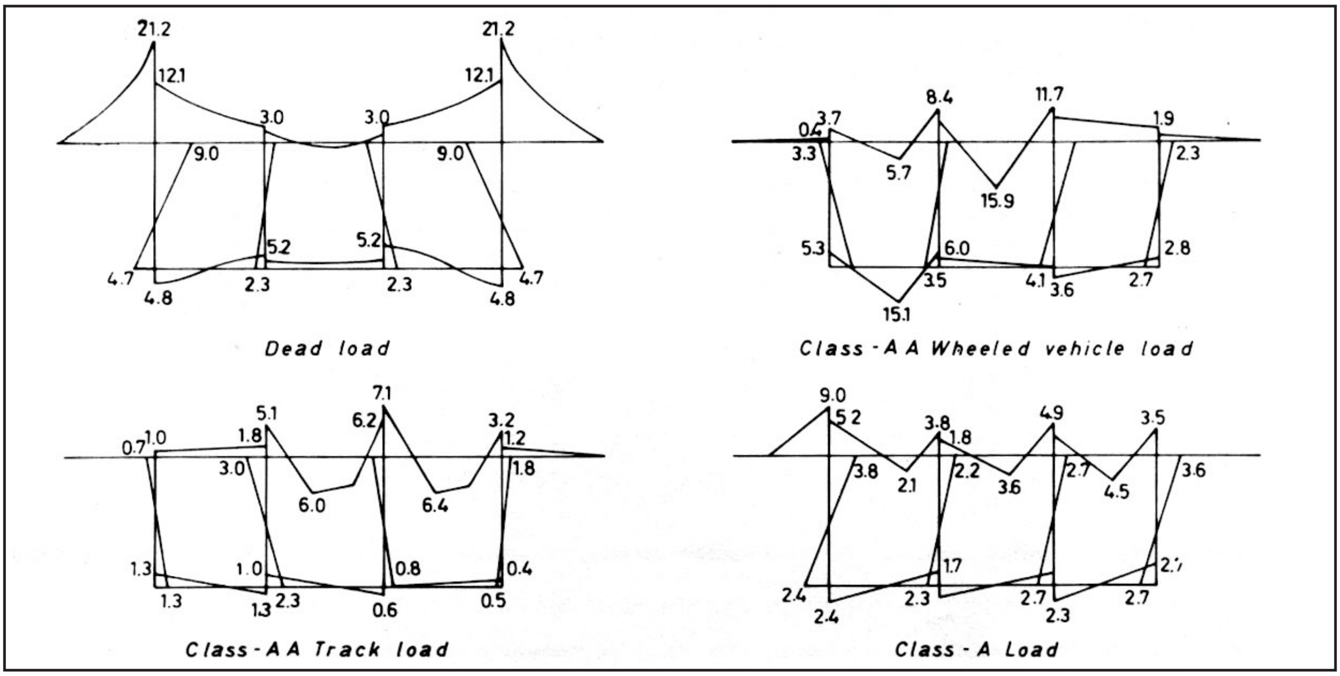

Figure 7. Variation in transverse moments for 25-m span

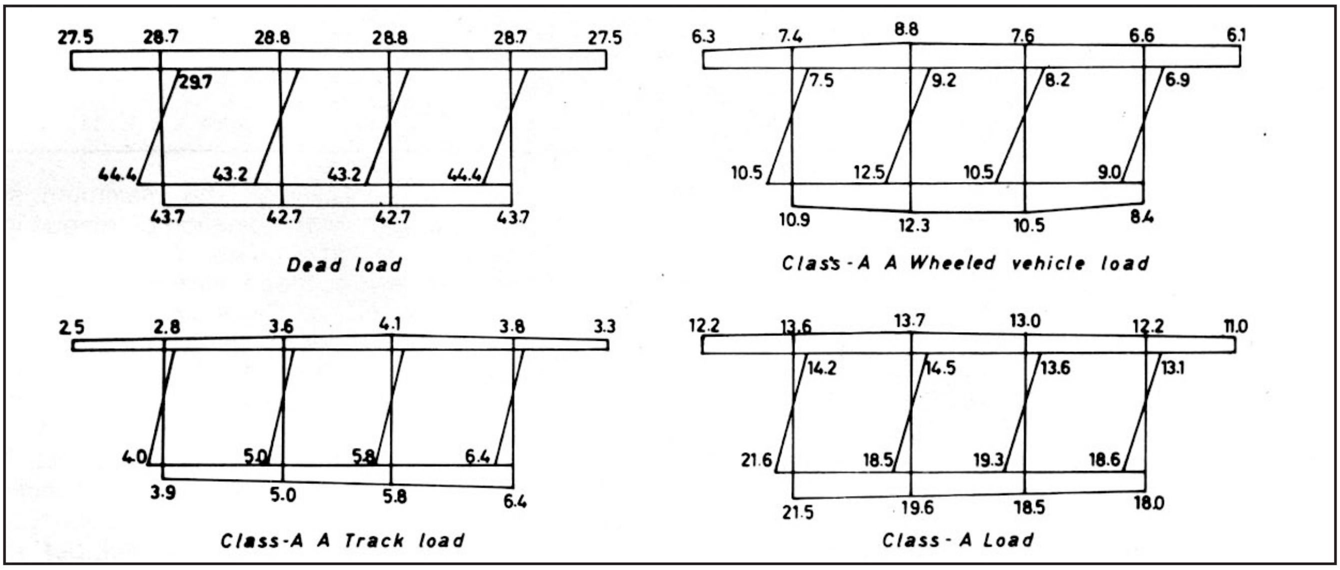

Figure 8. Variation in longitudinal stresses for 25-m span

Figure 9. One execution on DEC 2050 machine has taken 10 seconds of CPU time.

Cost estimation

The total cost per metre of span has been estimated by adding the prevailing market cost of concrete and steel. The cost of concrete has been taken as Rs. 800.00 per cubic metre with a cement bag weighing 50kg costing Rs. 65.00 and that of steel including placement as Rs. 5,100.00 per tonne.

Discussion and conclusions

Some of the important factors affecting the design and economy of the multicell cellular bridge decks are discussed in this section.

It is seen from the analysis that the system carries the loads primarily by developing longitudinal bending in top and soffit slabs, in-plane shear in the ribs and torsion in the cells. The decks need design, only for dead load and live load including impact. The effects due to other forces are minor and generally taken care of by the factor of safety and proper detailing. The maximum stresses, which can occur due to combination of various loadings, are taken as the design stresses.

The use of more than fifteen harmonics in the analyses does not improve the results appreciably. Therefore, in the analysis of various decks fifteen harmonics have been used. Figure 10 indicates that the optimum span for a three-cell cellular bridge deck adopted, is 25m. With this span the variation of forces in

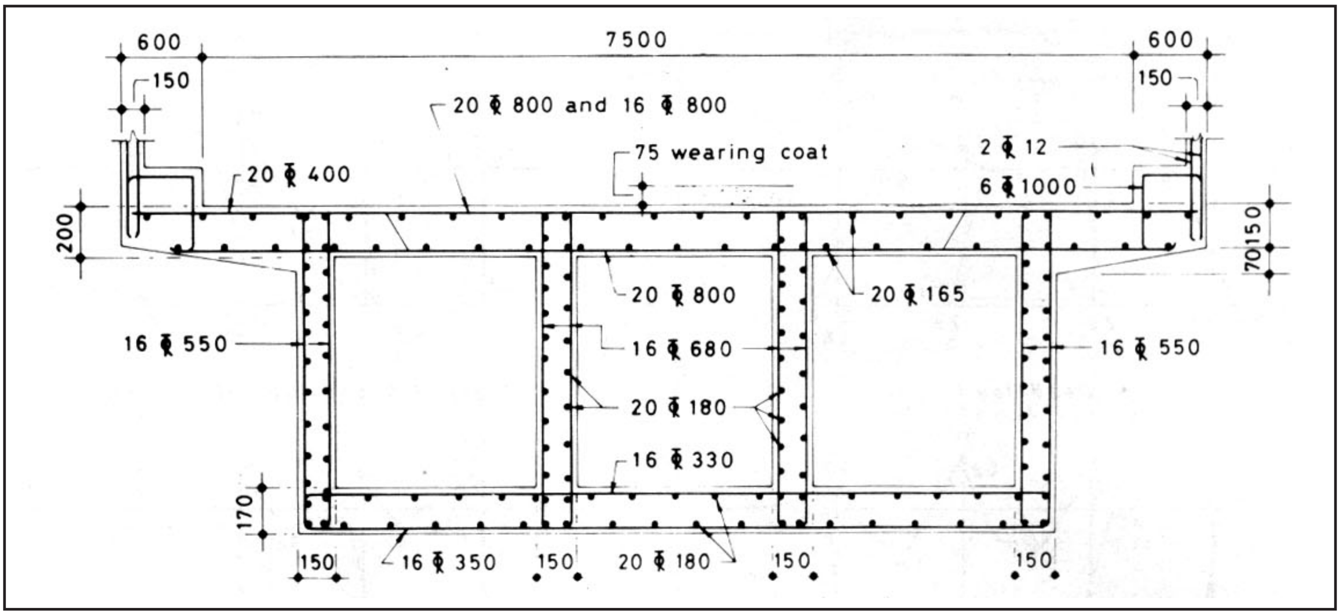

Figure 9. Reinforcement details for a typical cross- section of a 3-cell bridge deck for 25-m span

Table 2. Bending moments and stresses in various elements of bridge deck for different numbers of cells

Number of cells	Top slab				Softd slab			Longiludinal ribs			Coslim span Rs.
	Bending moment, kNm/m			Maximum longitudinal stress	Maximum bending moment, kNm/m		Maximum longitudinal stress	Maximum bending moment, kNm/m		Maximum longitudinal stress	
	Cantilever portion	Other portion									
		-ve	+ve	X10^{5}Nm^2	-ve	+ve	X10^{5}Nm^2	-ve	+ve	X10^{5}Nm^2	
2	31.3	22.1	16.9	42.0	6.9	4.8	66.8	10.8	5.9	67.6	6130.06
3	30.2	17.3	17.6	42.5	11.2	16.3	65.2	12.8	7.1	66.0	6885.18
4	30.9	17.7	11.2	42.6	8.0	8.6	63.5	13.6	8.4	64.3	9400.00

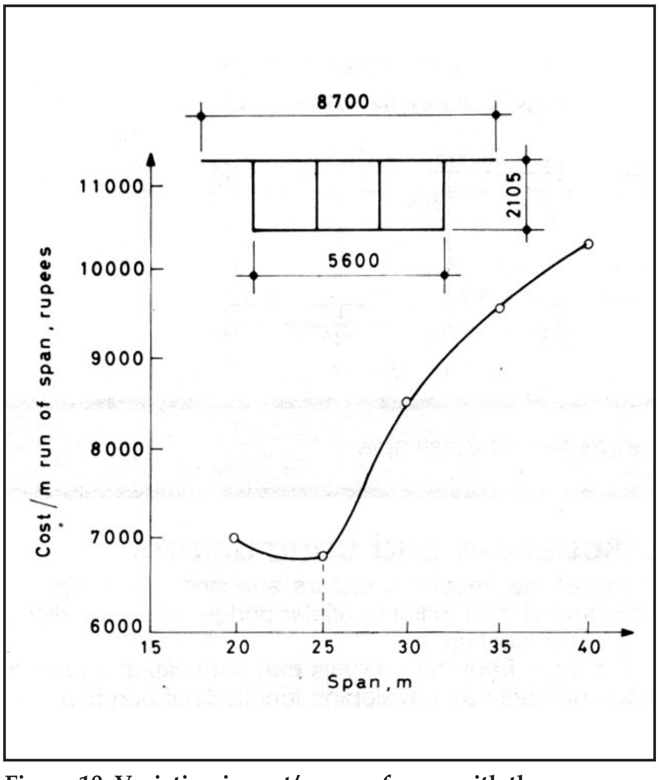

Figure 10. Variation in cost/m run of span with the span length

various components of deck and cost, with the number of cells is given in Table 2. The cost of the deck with two boxes is found to be minimum. It is seen that an increase in span beyond 25m results in a much higher increase in the cost of deck per metre run due to higher cost of additional longitudinal ribs; this increase is more than the stresses taken by them. The finite strip method is suitable for predicting the design stresses in the cellular bridge decks. The main disadvantage of the method is that it cannot be applied to decks with cells having varying thickness of walls.

References

1. CHEUNG, Y.K. Finite Strip Method in Structural Analysis. Pergamon International Library, 1976.
2. ---- Standard Specifications and Code of Practice for Road Bridges, IRC:/6-1966, Section II-Loads and Stresses. Indian Roads Congress. New Delhi, 1974.
3. SINGLA, K.G. Optimization of Cellular Bridge Decks, Thesis submitted for ME(Structures) Degree, Punjabi University, Patiala, 1984.
4. VICTOR. D. JOHNSON. Essentials of Bridge Engineering. Oxford and IBH Publishing Co., Third Edition, 1980.

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