Thermal stresses in non-prismatic concrete bridges for Indian summer conditions - 2

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The first part of the paper, published in the May 1990 issue of the Journal, discussed the behaviour on non-prismatic beams under thermal loading. The second and last part of the paper, presented here, deals with the behaviour of non-prismatic bridges subjected to the temperature distributions developed for the ambient conditions of New Delhi, India. Recommendations for the consideration of thermal stresses in the design of non-prismatic bridges are also included.

It is obvious from the discussion in the first part of the paper that compared to prismatic structures, the stresses at the mid-span of a multi-span structure increase and those at the intermediate support section of a two-span structure decrease as the beam section is varied along the span. The thermal curvatures, and thus, the thermal stresses in a structure depend upon the temperature distributions assumed in the analysis. While it is possible to draw generalised conclusions for linear temperature gradients, the same may not be quantitatively applicable to other non-linear temperature distributions. The self-equilibrating stresses under non-linear temperature distribution induce compressive stress at the beam soffit, which reduces the tensile stress due to the support moments. This necessitates investigations for specific temperature distributions.

The thermo-elastic analysis presented in this paper was carried out using the spline finite strip method[3]. The details of the method and of the thermo-elastic analysis of bridges are documented elsewhere and are not repeated here[3, 9]. It may be mentioned, however, that the web elements of varying depth were transformed into square domains by linear shape functions for developing the stiffness and load matrices. The stiffness and load matrices of the web and the bottom flange were transformed to global co-ordinates before assembling into the global matrices.

The ratio K was varied from 1.00 (prismatic structure) to 3.0 in steps of 0.5; the usual range of K is upto about 2.0 in practice, but values of about 3.0 are not uncommon[10]. The web thickness was varied linearly from 0.5m to 0.8m in nonprismatic structures of constant depth. The thickness of the bottom flange, where applicable, was varied from 0.18m to 0.4m. It is very difficult to generalise these variations but the values chosen represent the current design trends in medium and lona-soan structures.

The depth d_2 of the structure was varied from 1.0m to 4.0m and the length of the span was assumed to be 40.0m for computational purposes, although thermal stresses in bridges are independent of the span length. The structure was generally divided into 30 elements in the longitudinal direction. In the transverse direction, the box girder was divided into 12 elements making use of symmetry, and the T-beam into 7 elements; the web was taken as one element in the transverse direction. The integration for stiffness and load matrices was carried out using Gaussian quadrature points.

The temperature distributions developed for New Delhi ambient conditions were employed in the analysis[1]. It was assumed in the analysis that the same were also applicable for the maximum stress condition in non-prismatic structures. The change in the soffit tensile stress σ_t always refers to that compared to a prismatic beam of the minimum section of the non-prismatic beam, i.e. that corresponding to the midspan section in multi-span and to the end support in two-span structures.

Young's modulus of concrete was assumed to be 35 GNm^{2} and the coefficient of thermal expansion of concrete to be 10.8 x 10^{6} K^{1} in all the computations presented in this paper.

Multi-span structures

T - beam bridges: The soffit tensile stresses a t at the midspan section of an intermediate span are indicated in Figure 6 for T-beams of variable depth. It is obvious from Figure 6 that any increase in the beam section increases the mid-span soffit tensile stresses σ_t. The increase in σ_t, was found to be greater for the linear soffit than that for the parabolic soffit profile. The stresses increased rapidly at first as K was increased, but the increase in σ_t got progressively smaller as K was increased further. The increase in σ_t also depended upon the depth d_2 at the end support. In the case of linear soffit profile, σ_t increased by about 57 percent for K = 1.5 when d_2 = 1.0m and 1= 0.5; the corresponding increase in at for K = 2.0 was about 100 percent and that for K = 3 .0 was about 163 percent. For parabolic variation of depth, the corresponding increases were 34 percent for K = 1.5, 53 percent for K = 2.0 and 106 percent for K = 3.0. The increase in α_t for the beam with d_2 = 4.0m was relatively smaller when compared to that in a beam of d_2 = 1.0m, being about 45 percent for K = 1.5, 82 percent for K = 2.0 and 133 percent for K = 3.0 for linear variation of the depth and y = 0.5. For parabolic variation, the increase in a t for d_2 = 4.0m was about 29 percent for K = 1.5, 57 percent for K = 2.0 and 64 percent for K = 3.0.

For the usual value of K = 2.0, it can be said that the increase in a t varies from about 100 to 82 percent as the minimum depth of the section is varied from 1.0m to 4.0m for the linear soffit profile, and for the parabolic soffit profile the increase is around 55 percent.

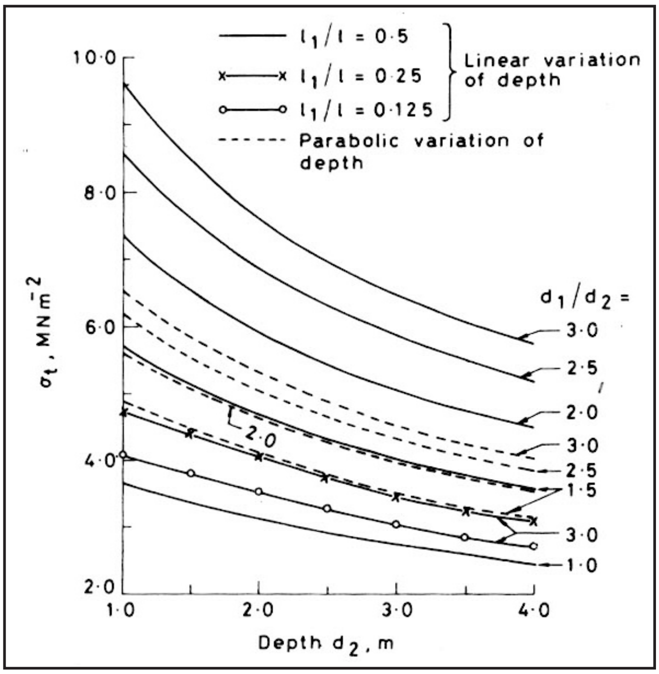

Figure 6. Variation of mid-span soffit stresses in multi-span continuous non-prismatic T-beams

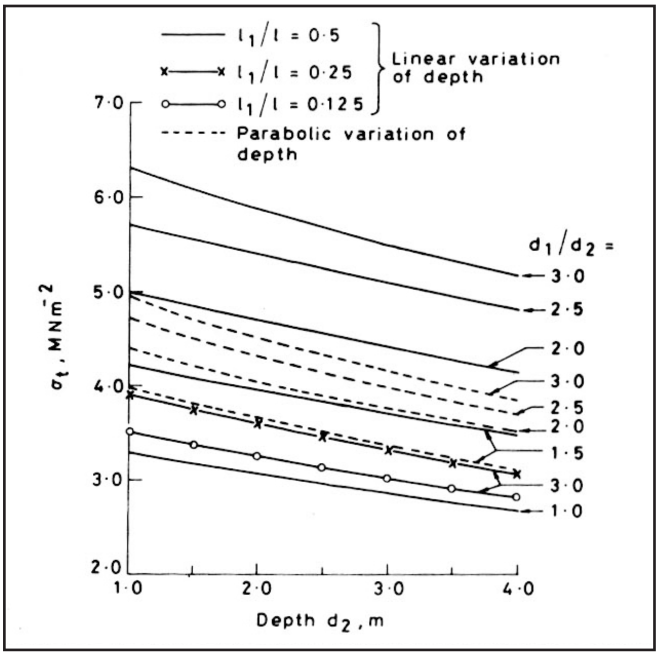

Figure 8. Variation of mid-span soffit stresses in multi-span continuous non-prismatic box girders

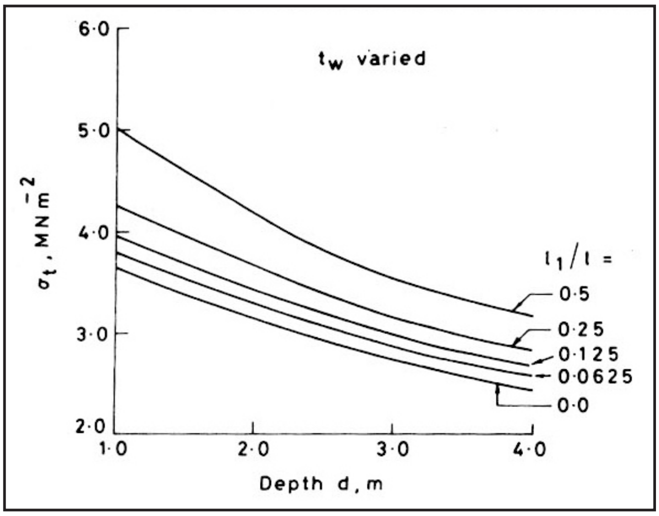

Figure 7. Variation of mid-span soffit stresses in multi-span continuous non-prismatic T-beams of constant depth

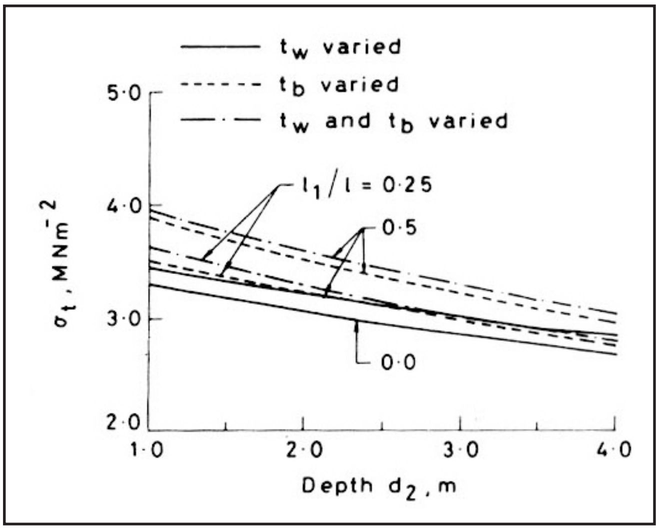

Figure 9. Variation of mid-span soffit stresses in multi-span continuous non-prismatic box girders of constant depth

However, when l_1 was reduced, the increase in at was much smaller. For Y = 0.25 for instance, the increase in σ_t , was less than 30 percent and for Y= 0.125 the increase was found to be barely 10 percent over the entire range of the depths considered. even for K = 3.0, Figure 6.

As explained earlier, the increase in σ_t was comparatively smaller in non-prismatic beams of constant depth. The values of σ_t in T-beams of constant depth as the web thickness is varied from 0.5m to 0.8m are shown in Figure 7. The increase in a t varied from 37 to 28 percent as the depth of the beam was varied from 1.0m to 4.0m for Y = 0.5. For smaller values of Y the increase in a t was found to be much less, varying from about 15 percent for Y = 0.25 to about 5 percent for Y = 0.0625.

The increase in α_t was thus much less significant when the web thickness was varied than when the beam depth was varied over the same length of the beam. Further, if the variation in the cross-section was over a length less than about 1/8th of the span, the increase in α_t appeared to be negligible.

Box girder bridges: The response of box girders with ventilated and unventilated air-cells was found to be similar except that α_t was smaller in the box girders with ventilated air-cells. Thus only the results of the box girders with unventilated air-cells are presented in this paper: The values of α_t in the mid-span region of the box girders for various values of K, and linear and parabolic soffit profiles are shown in Figure 8. The increase in α_t was found to be much smaller in box girders than that in T-beams. α_t increased by about 30 percent for K = 1.5, 55 percent for K = 2.0 and 92 percent for K = 3.0 over the entire range of the depths considered when y = 0.5 and the soffit profile was linear. The corresponding increase in α_t for the parabolic soffit profile was found to be about 15 percent for K = 1.5, 30 percent for K = 2.0 and 50 percent for Pc= 3.0. The increase in α_t was found to be slightly smaller for larger values of depth.

However, the increase in σ_t was found to be much smaller when the girder depth was varied over a smaller length (l_1). These plots are also included in Figure 8. The increase in α_t was found to be less than 20 percent for Y = 0.25 and that for Y= 0.125 to be around 5 percent when K. = 3.0. For smaller values of K , the increase in α_t was almost negligible.

The values of α_t computed for constant-depth, non-prismatic girders are plotted in Figure 9. Three cases were considered, namely t_w, varied, t_b varied, and both t_w and t_b varied. The increase in σ_t was found to be barely more than 4 percent when t_w alone was varied over the entire span. However, α_t was more sensitive to the variation in t_b; the increase in at was upto 17 percent when Y = 0.5 and about 5 percent when y was reduced to 0.25. If both t_w and t_b were varied, the increase in a, was found to be upto about 20 percent for y= 0.50 and about 10 percent for Y= 0.25. For still smaller values of Y, the increase in α_t was further reduced, and hence, almost negligible. It should be noted here that Y= 0.0 implies a prismatic beam.

Two-span structures

The thermal response of two-span structures or the end spans in multi-span beams will be much different from that of an intermediate span of a multi-span bridge elucidated earlier. The ranges of the various parameters considered for two-span structures were the same as those for the multi-span bridges.

T-beam bridges: The soffit tensile stresses computed at the intermediate support section of two-span T-beam bridges are shown in Figure 10. It can be seen that a t decreases rapidly as K is increased from 1.0. Further, the decrease is more significant for the parabolic soffit profile than that for the linear soffit profile. The decrease in at for the linear soffit profile was found

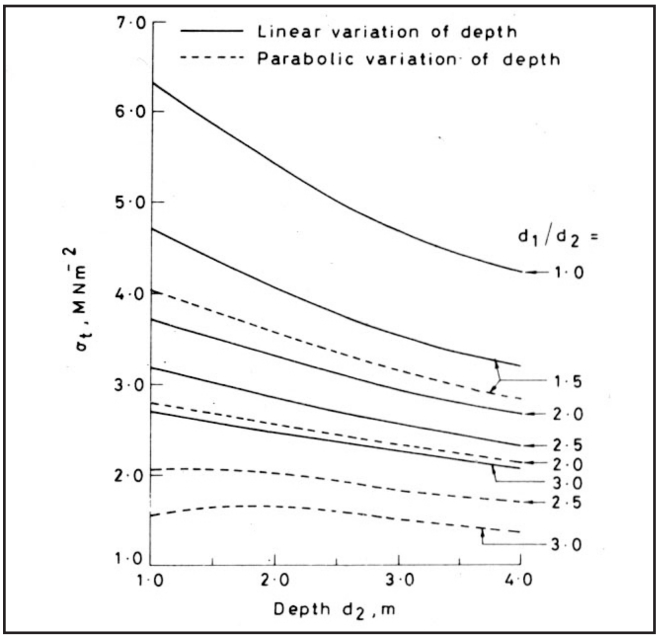

Figure 10. Variation of soffit stresses at the intermediate support section of two-span continuous non-prismatic T-beams

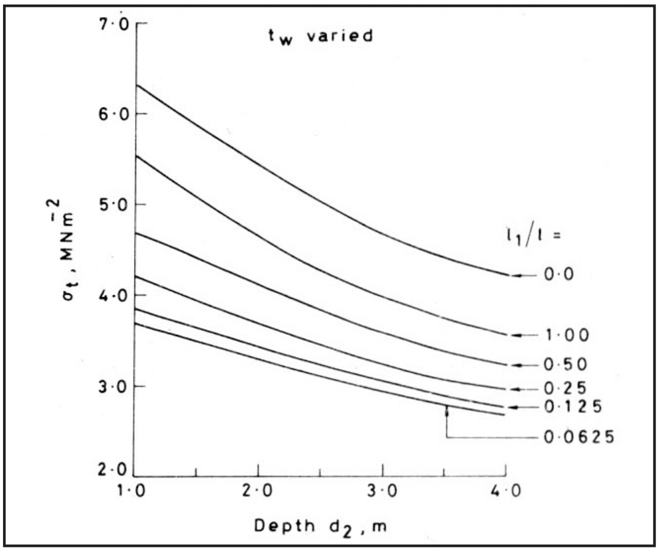

Figure 11. Variation of soffit stresses at the intermediate support section of two-span continuous non-prismatic T-beams of constant depth

to be about 25 percent when K= 1.5, 40 percent when K = 2.0 and 55 percent for K = 3.0. In the case of the parabolic soffit profile, the decrease in σ_t was about 35 percent for K = 1.5, around 50 percent for K = 2.0 and upto 75 percent for ec = 3.0. If the beam depth was varied over a smaller length, the decrease in σ_t was much greater; α_t was reducea to less than a quarter when k = 2.0 and y was less than 0.25. These values are, however, not indicated in Figure 10.

In the case of beams of constant depth but varying t_w, the reduction in α_t was found to be relatively less significant,

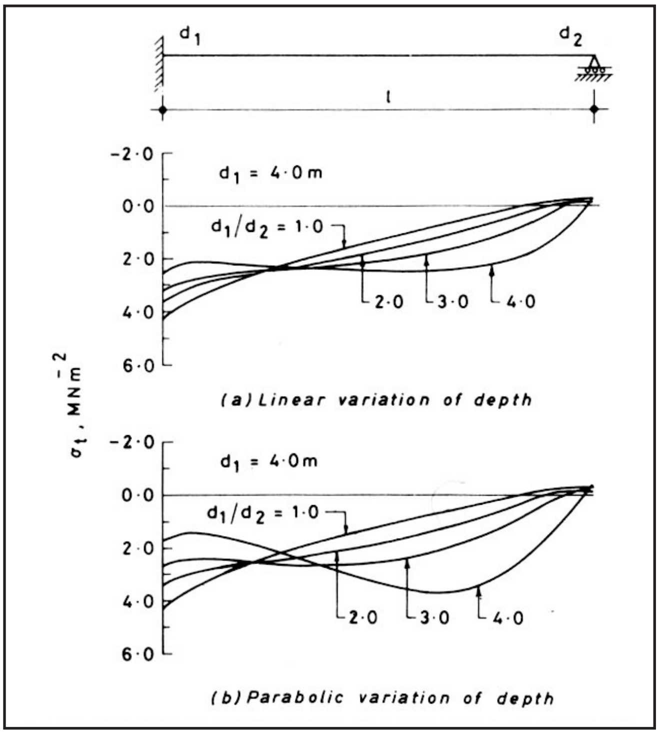

Figure 12. Variation of soffit stresses along the span in two-span continuous non-prismatic T-beams

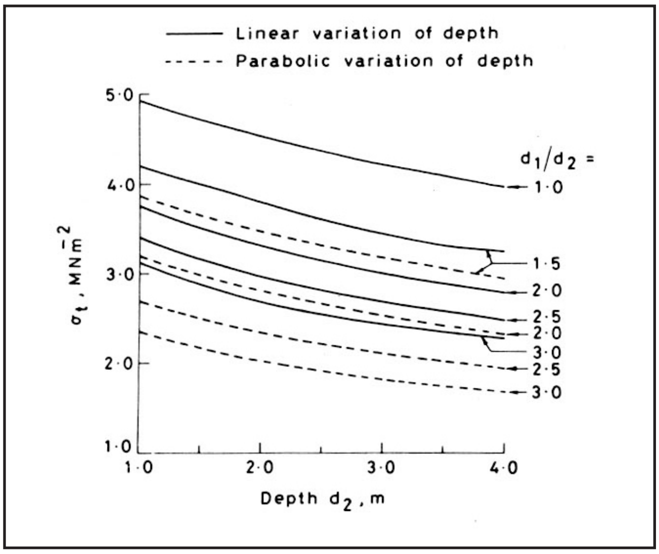

Figure 13. Variation of soffit stresses at the intermediate support section of two-span continuous non-prismatic box girders

Figure 11. The reduction in α_t was more significant, as y was reduced from 1.0; the reduction being upto 15 percent for Y= 1.0, upto 25 percent for Y= 0.5 and upto 40 percent for Y= 0.0625.

It must be noted, however, that the reduction in α_t at the support section will be accompanied by an increase in α_t in the mid-span region. The variation of σ_t along the span in T-beams of d_1 = 4.0m is plotted in Figure12. The maximum soffit tensile stress σ_t is shown for linear soffit profiles in Figure 12 (a) and for parabolic soffit profiles in Figure 12 (b). It can be observed that σ_t is reduced at the intermediate support section as K is increased, but α_t increases significantly in the mid-span region. In the extreme case of K= 4.0, σ_t in the mid-span region is much greater than that at the support section in the case of parabolic soffit profile. Even in the case of K= 2.0, the increase in σ_t at the mid span section is a little over 30 percent for the linear profile and 50 percent for the parabolic profile. The increase in σ_t at the mid-span section was generally found to be more significant for parabolic profiles than for linear profiles.

Similarly, in the case of non-prismatic beams of constant depth, if the section is varied over a short length near the support, α_t at sections away from the support may be higher than that at the support section and should be checked in the design process.

Box girder bridges: The variation of the soffit tensile stresses in two-span continuous box girders is indicated in Figure 13. The

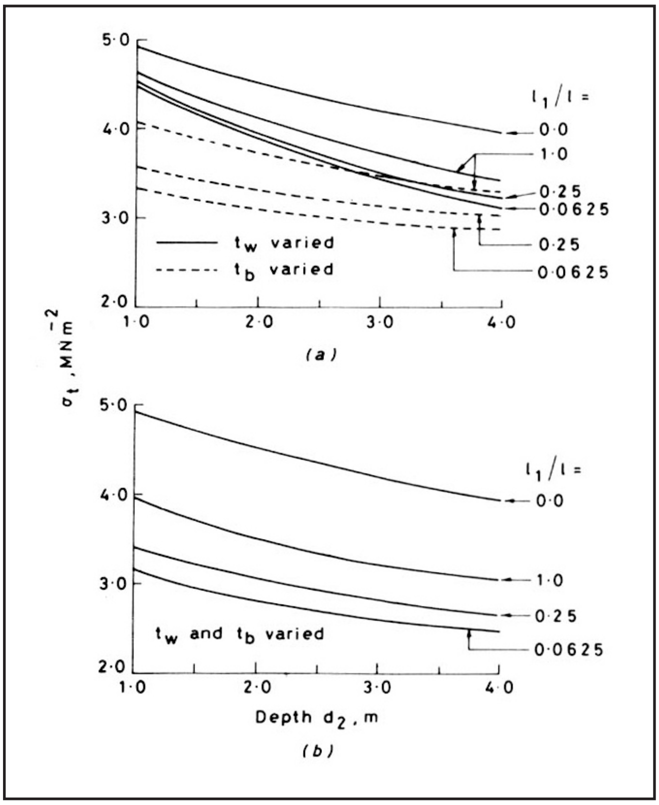

Figure 14. Variation of soffit stresses at the intermediate support section of two-span continuous non-prismatic box girders of constant depth

reduction in α_t was generally found to be smaller than that observed in the T-beam but the general trends were the same.

The reduction in α_t for linear soffit profiles was found to vary from about 14 to 17 percent as the depth d_2 was increased from 1.0 to 4.0m when K= 1.5; the corresponding reduction for K= 2.0 was found to vary from 24 to 30 percent and for K= 3.0 from 36 to 43 percent. For parabolic soffit profile, the reduction in σ_t when K = 1.5 was found to vary from about 21 to 26 percent as d_2 was varied from 1.0 to 4.0 m; the corresponding reductions for K = 2.0 being from 35 to 42 percent and for K= 3.0 the reduction varied from 52 to 58 percent. The reduction in σ_t is thus slightly more for larger girder depths and more significant for parabolic soffit profiles than for linear profiles. Even in this case it was observed that σ_t decreased much more when Y was reduced from 1.0.

Figures 14 (a) and 14 (b) show the variation of α_t in non-prismatic box girders of constant depth. The variation of σ_t as t_w and t_b are varied separately, is indicated in Figure 14 (a), whereas Figure 14 (b) indicates the variation of σ_t when t_w and t_b are varied together. The values are plotted as y is varied from 1.0 to 0.0625. It can be observed from Figure 14 (a) that α_t decreases more rapidly when tb is varied than when t_w is varied over the same length. The reduction in at (when t_w was varied) was found to be about 6 to 9 percent as y varied from 1.0 to 0.0625 for 1.0-m deep girder. The corresponding reductions for 4.0-m deep girder were found to be about 14 to 21 percent. When t_b was varied the reduction in o t was found to be about 18 percent for y= 1.0 and about 30 percent as Y was reduced to 0.0625. Similarly if both t_w and t_b were varied together, the reduction in a t was found to be slightly more than that for the cases examined earlier, Figure 14 (b). It can be noted from

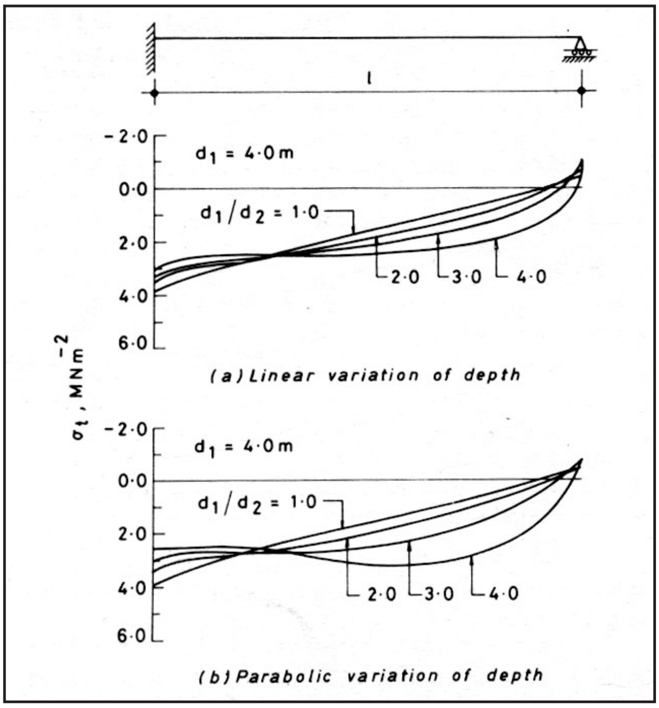

Figure 15. Variation of soffit stresses along the span in two-span continuous non-prismatic box girders

Figure 14 (b) that a t decreases by about 20 percent for 11 = 1.0 and by about 35 percent when y = 0.0625.

However, it is prudent to check for α_t at sections away from the intermediate support as well. The variation of σ_t along the span in box girders of d_1 = 4.0m indicates an increase of a, in the mid-span region, Figures 15 (a) and 15 (b). It is obvious that though a, decreases at the intermediate support section, it increases at other sections along the span, the increase being more pronounced for parabolic than for linear profiles. In the extreme case of K = 4.0, α_t at the mid-span section is nearly the same as that at the support section for the linear soffit profile and nearly 30 percent more than that at the support section for the parabolic soffit profile, whereas generally σ_t at the mid-span section is less than half the value at the support section in prismatic structures. For the more usual case of K = 2.0, σ_t was found to increase by about 25 percent at the midspan section.

Discussion

Thermal response of non-prismatic structures is much different from that of prismatic structures and depends upon a number of factors — mainly upon geometry of the structure, temperature distribution, and support conditions. It is shown that the thermal moments induced in the structure vary significantly depending upon the temperature distribution assumed.

Closed-form solutions presented in this paper for the simple cases of rectangular prismatic beams for a linear temperature distribution, though of little practical application, provide insight into their thermal response. Based on such models, it is shown that the thermal stresses generally increase in multi-span structures and decrease in two-span structures. The stresses in multi-span rectangular beams for the linear temperature distribution (LTD) increase by about 85 percent for the usual value of K = 2.0. In a T-beam for the NDTD the corresponding increase in α_t was found to be upto about 100 percent for the linear soffit profile and about 53 percent for the parabolic soffit profile. The corresponding increases in box girders being only about 55 percent and 30 percent for linear and parabolic soffit profiles, respectively.

The depth of a beam with parabolic profile at any intermediate section is smaller than the corresponding section of a beam with linear profile for the same mid-span and support section depths, when the depth is varied over the same length. Thus, the thermal loading for parabolic soffit profile will be smaller than that for the corresponding linear profile, which induces smaller thermal moments and thus smaller stresses.

In the case of two-span structures, the maximum section at the support is also the section subjected to maximum moment in the span. Though the thermal moments are increased due to increased thermal loading in non-prismatic structures, the stresses at the support section will be smaller, as the relative increase in the section modulus will be generally more than the increase in the thermal moments. For instance, the stresses in rectangular non-prismatic beam decrease by about 25 percent for K = 2.0 and Y =1.0 compared to a prismatic beam. In T-beams for the NDTD, the corresponding reduction in stresses was found to be about 40 percent for the linear soffit profile and about 50 percent for the parabolic profile. Similarly, for box girders the reduction in the stresses for the linear soffit profile was found to vary from 24 to 30 percent, and for the parabolic soffit profile the reduction was found to vary from 35 to 42 percent, depending upon the girder depth when K = 2.0.

Further, it was also observed that as l_1 was reduced the mid-span stresses were reduced. It is thus beneficial from thermal stress considerations to keep l_1 as small as possible, which is in consonance with the usual design practice.

However, it should be noted that the soffit tensile stresses may be critical at the intermediate support section of two-span prestressed concrete bridges. The compressive stress at that section under self-weight condition may be extremely small and the tensile stress due to temperature differential may be substantial enough to cause cracking. These stresses are likely to decrease in non-prismatic bridges. However, it will be necessary to check for the thermal stresses at the midspan. Region as well, since the soffitt tensile stress due to thermal and other superimposed loads are additive in the mid-span region.

Nevertheless, it may be too conservative to combine the thermal effects with full live load stresses, since the probability of full thermal load and full live load acting together is not very great. It may be adequate to adopt the German and the French practice of designing for the combinations of full live load and part thermal load or part live load and full thermal load[2,5]. However, the factors for combining the thermal and superimposed loads need to be determined through further investigations.

Conclusions and recommendations

The following conclusions may be drawn from the analyses and discussion presented in this paper.

1. the soffit tensile stresses in multi-span non-prismatic structures are greater than those in prismatic structures, the increase depending upon the geometry of the structures. The increase in stresses for the temperature distributions developed for New Delhi ambient conditions can be as much as 100 percent for T-bridges when the support section depth is twice that at the mid-span and the depth is varied linearly; the increase for box girders is relatively smaller being about 55 percent for the same parameters. The increases will be much smaller for parabolic soffit profiles — about 50 percent for T-beam and 30 percent for box girder structures when c = 2.0. The increase in stresses is relatively much smaller in non-prismatic structures of constant depth.

2. The increase in thermal stresses in multi-span structures will be smaller if the length over which the cross-section varies, is reduced. If this length is less than about 1/8th of the span, the increase in thermal stresses is almost negligible.

3. Thermal stresses in the intermediate support region of two-span non-prismatic structures are reduced compared to those in a prismatic structure of the same depth as the end support section. The reduction is again dependent upon the geometry of the structure and is relatively greater for the parabolic soffit profile than that for the linear profile for the same change in the structural depth. The reduction is relatively smaller in non-prismatic structures of constant depth than in those of variable depth.

4. The smaller the length of the change in cross-section, the greater will be reduction in the thermal stresses at the intermediate support section of two-span structures. However, the thermal stresses at other sections should be checked.

5. It will be conservative to combine thermal effects with other loads. It may be adequate to combine full thermal load with part live load or part thermal load with full live load. These factors have to be determined from the data of actual live loads on bridges and the variation of ambient temperatures.

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