Thermal stresses in non-prismatic concrete bridges for Indian summer conditions - 1

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Most of the publications on thermal stresses in concrete bridges pertain to prismatic structures. The behaviour of non-prismatic beams under thermal loading is discussed in the first part of the paper presented here. Simplified models of non-prismatic beams analysed for thermal stresses herein provide insight into their behaviour. The second part of the paper deals with the behaviour of non-prismatic bridges subjected to the temperature distributions developed for the ambient conditions in New Delhi, India.

Temperature distributions for the consideration of thermal stresses in concrete bridges based on the ambient conditions in New Delhi, India, were presented in an earlier paper[1]. The studies available on thermal stresses in concrete bridges pertain mostly to prismatic structures. However, bridge structures are seldom of constant section throughout the length as the cross-section is usually varied in order to accommodate changes in the bending moment or shear force, especially in the region of supports. Thermal stresses in a non-prismatic structure differ significantly from those in a prismatic structure, because they depend upon the geometry of the structure as well[2].

The variation of thermal stresses in non-prismatic concrete bridges of T- and box-sections for the New Delhi Temperature Distributions (NDTD) is discussed in this paper[1]. Two-span and multi-span structures of variable depth as well as those of constant depth but of varying cross-section were included in these investigations, Figure 1. It is shown that the thermal stresses increase in an intermediate span of a multi-span continuous bridge, but decrease in two-span structures as the bridge section is varied along the span. Linear and parabolic variations of the depth along the span were considered. The variation of the depth along the entire span and in a part of the span was considered in the case of linear soffit profile, Figure 1(a) and 1(b). In the case of constant depth structures, the

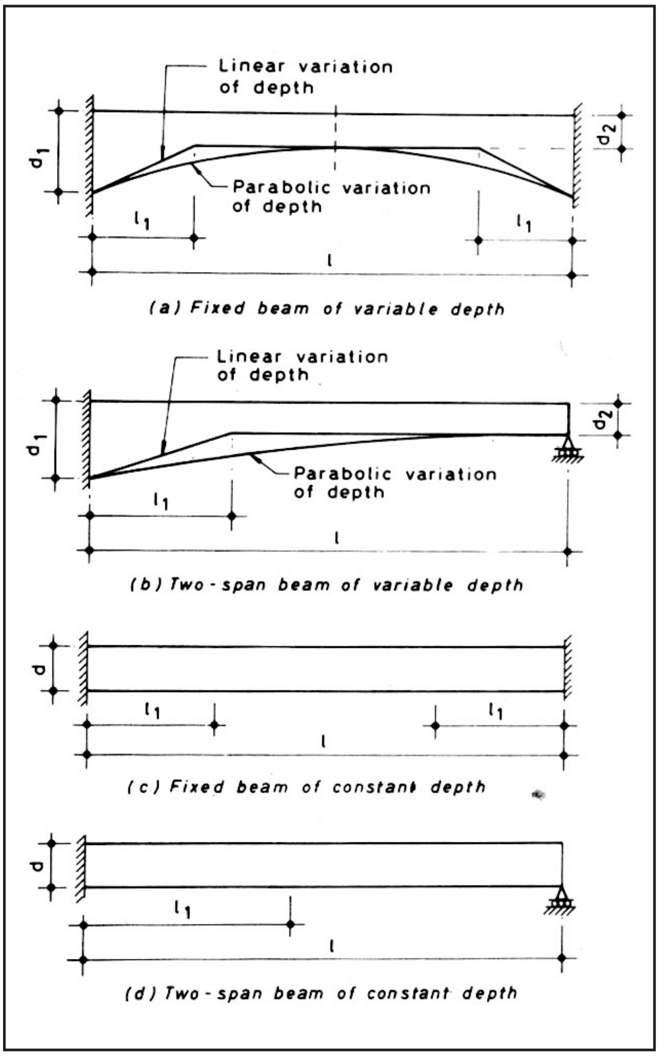

Figure 1. Non-prismatic beams
(t = span, t_1 = length of variable section)

thicknesses of the web and the bottom flange were varied in the support regions as indicated in Figure 1(c) and 1(d).

Closed-form solutions are presented for the thermo-elastic analysis of non-prismatic beams of rectangular sections under linear temperature differentials. Thermal stress variation in non-prismatic structures is explained with the help of these models. Further analyses of more complex T- and box-sections were carried out using the Spline Finite Strip Method[3].

Non-prismatic structures

A bridge structure is seldom of constant section. The current trend of bridge design is to keep the depth of the section nearly constant and to vary the web and the flange thicknes ses in order to reduce the construction time and ,costs. Nevertheless, structures of variable depth were also included in the present study. The depth of the structures is usually varied either parabolically or linearly along the span, such that the maximum depth is available at the intermediate supports of a continuous bridge. Possible variation of the depth in an intermediate span of multi-span continuous beam is indicated in Figure 1(a) and that of one span of a two-span continuous structure in Figure 1 (b). In the case of linear soffit profile, the depth of the structure may be varied in a part of the span rather than in the entire span. The length over which the cross-section varies is shown as l_1 in Figure 1. A ratio of $(l_1/l) = 0.0$ indicates a prismatic structure. Structures of constant depth but of variable web thickness (t_w)or bottom flange thickness (t_b) are shown in Figure 1(c) and 1(d). Only the bottom flange thickness t_b is varied in these investigations as this is more common practice. The cross-sections of the bridges considered in this paper, viz. box-section, T and rectangular beams, are indicated in Figure 2. The width of the box girder is adequate to accommodate two-lane carriageway with footpaths on both sides as per the Indian Roads Congress (IRC) specifications[4]. The T-section indicated in Figure 2 can be taken as a part of a multi-beam bridge.

Behaviour under thermal loading

Most of the publications on thermal analysis of bridges assume the structure to be prismatic for analytical simplicity. Thermal stresses in non-prismatic bridges of box-section under linear temperature differential were presented by Podolny[2]. It was shown that the stresses in the intermediate span of a multi-span non-prismatic continuous bridge increase by more than 50 percent compared to prismatic structures in the usual range of the variation of depth. However, the analysis pertained only to the parabolic variation of depth and a linear temperature differential. The temperature differentials in bridges are usually non -linear, and thus, the variation of thermal stresses is likely to be different. An intermediate span of a multi-span continuous bridge can be assumed to be a beam with both ends restrained against rotation and vertical displacement, but free to deform longitudinally. Similarly, a two-span continuous bridge can be analysed as a propped cantilever with the intermediate support assumed to be encastre and the end support free to deform longitudinally on account of the symmetry of loading and the boundary conditions.

Notation

b	...	width of the section
b_1, b_2	...	maximum and minimum widths of the beam
d	...	depth of the beam
d_1, d_2	...	maximum and minimum depths of the beam
E_c	...	Young's modulus of concrete
f_1, f_2	...	functions defined
I	...	moment of inertia of the beam
I_1, I_2	...	maximum and minimum moments of intertia of the beam
k	...	a coefficient
l	...	span of the beam
l_1	...	length over which the beam section varies
m	...	longitudinal thermal moment
m_1	...	longitudinal thermal moment in a beam of unit depth or of constant depth
T	...	top-surface temperature
t_w	...	web thickness
t_b	...	bottom flange thickness
$\propto$	...	coefficient of thermal expansion of concrete
β	...	(b_1/b_2)
β_1	...	$(\beta - 1)$
γ	...	(l_1/l)
κ	...	(d_1/d_2)
κ_1	...	$(\kappa - 1)$
σ_t	...	maximum soffit tensile stress

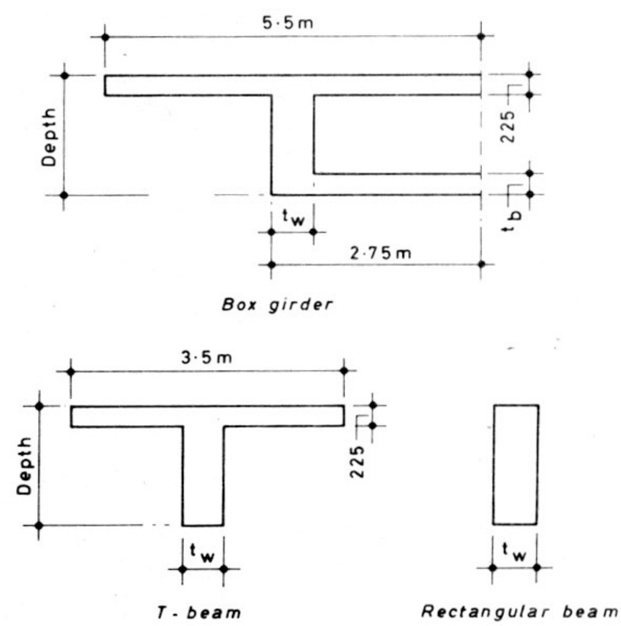

Figure 2. Cross-sections considered in the analysis (t_w = 0.5 m and tb 0.18 m when not varied; in other cases t_w varies from 0.5 to 0.8 m and tb varies from 0.18 to 0.40 m)

It is obvious that the thermal stresses in a structure depend to a large extent on the temperature distributions assumed in the analysis. The temperature differentials cause self-equilibrating stresses as well as support moments in a continuous structure. The support restraining moments depend upon the structural geometry and the thermal curvatures developed in the structures. The thermal curvatures vary considerably as the section is varied. The thermal moment m developed in a structure can be expressed as

$$m = k E_c I \psi \qquad \text{... (1)}$$

where,

m	=	thermal moment
k	=	a coefficient
E_c	=	Young's modulus of concrete
I	=	moment of inertia of the section
ψ	=	thermal curvature of the section.

The coefficient k depends upon the support conditions. The variation of thermal moments in an intermediate span of a prismatic rectangular beam as a ratio of the moment rn, in a 1.0-m deep beam is shown in Figure 3 for various temperature distributions. The linear temperature distributions of the French and the German codes (LTD)[2,5] and the non, linear distributions of New Zealand (NZTD)[6], Australia (NAASRA)[7], as well as those developed for Melbourne, Australia (PMTD and NMTD)[8] and New Delhi, India (NDTD)[1] were included in these computations. It can be noticed that there is a vast difference in the thermal moments developed in the beam for various distributions. The moments developed for the LTD and NMTD are nearly in the same proportion as the moments of inertia of the sections, whereas in all other cases, the increase in the thermal moments is much smaller than that of the moment of inertia. The thermal curvature in a non-prismatic structure varies along the span and causes higher support moments compared to a prismatic structure of the minimum beam section. The thermal response of a non-prismatic beam will thus depend upon its geometry and the temperature distribution considered among other parameters.

Multi-span continuous rectangular beam

As a simplified model, thermal moments in a non-prismatic rectangular beam for linear temperature differential were computed. Closed-form solutions can be obtained for such cases by treating the support moments as the redundants and by imposing compatibility conditions at the supports. The moment in a beam of varying depth can be shown to be

$$m = E_c L_2 \left(\frac{r \propto}{d_2}\right) \left[\left\{\frac{2\, y\, l_2\, k\, (t-2\, y)\, k_1}{y(k+1)+(t-2\, y)\, k^2}\right\}\left(\frac{k^2}{k_1}\right)\right] \qquad \text{...(2)}$$

where,

d_1	=	depth at the support
d_2	=	depth at the midspan region
l_2	=	moment of inertia of the mid-span section
l	=	span of the beam
t_1	=	length over which the beam section is varied
y	=	(l_1/l)
K	=	(d_1/d_2)
K_1	=	(k-1)

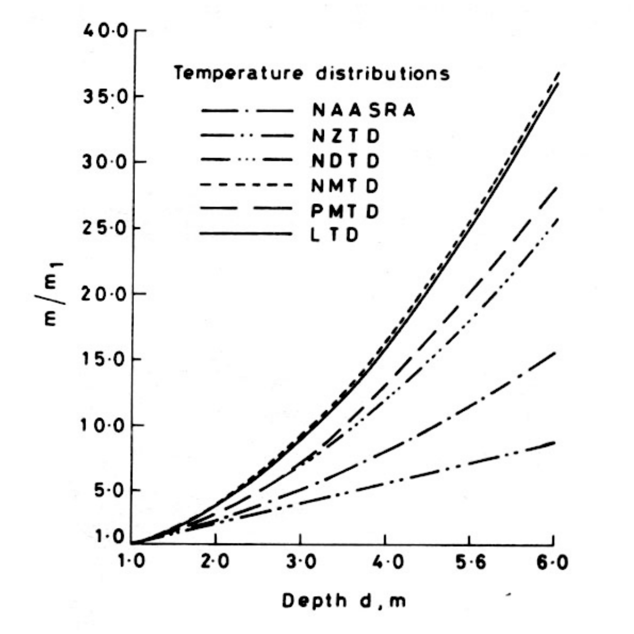

Figure 3. Variation of the ratio of thermal moments (m/m_1) with the depth of rectangular beams for various temperature distributions

T	=	temperature at the top surface
α	=	thermal coefficient of expansion.

The variation of the ratio (m/m_1) for this beam for various values of y as a function of K is shown in Figure 4(a). In this case, m_2 is the thermal moment in a prismatic beam of depth d_2. It can be seen that the moments in the beam increase significantly as the ratio K is increased. However, increase in the moment is not very significant for values of y smaller than about 0.20 regardless of the value of K.

Similarly the moments in a beam of constant depth *d* but of thickness varying linearly over a length u near the supports can be shown to be

$$m = E_c L_2 \left(\frac{r \propto}{d_2}\right) \left[\frac{\beta}{2\, y\, l_n\, \beta+(1-2y)\, \beta_1}\right] \qquad \text{...(3)}$$

where,

β	=	(b_1/b_2)
β_1	=	(β – 1)
b_1	=	thickness at the supports
b_2	=	mid-span thickness

The values (m/m_1) are plotted against β for various values of y, Figure 4(b). The response of the beam to the variation in the geometrical parameters was similar to that of the beam of varying depth, but the increase in the moments was relatively much smaller. This is because the thermal loading on the beam increases much more rapidly when its depth is increased than

(LTD = Linear temperature distribution, NZTD = New Zealand temperature distribution, NAASRA = Australian specifications, PMTD and NMTD = proposed temperature distributions for Melbourne, NDTD Proposed temperature distributions for New Delhi) when its thickness is increased. However, increase in the moments in this case was found to be higher than that due to a similar increase in the value of K for y less than 0.30.

This simple model provides insight into the thermal response of non-prismatic structures. As the support section is increased, the thermal moments in the structure increase. Since the moments in an intermediate span of a multi-span continuous structure are constant along the span, the mid-span section — being the smallest — will be subjected to increased stresses compared to a prismatic structure. The model also indicates that the increase in the stresses is more significant in beams of varying depth than in those of varying thickness; further, the moments, and thus, the stresses increase as l_1 is increased.

For instance, for the case of K = 4.0 the moment in a non-prismatic beam increases 3 times compared to that in a prismatic beam for Y = 0.5, but only 1.4 times for Y = 0.3, Figure 4(a). Thus, the mid-span stresses in the non-prismatic beam increase by the same factors compared to a prismatic beam. Similarly, for K = 2.0 the stresses increase 1.85 times when Y= 0.5, but only 1.3 times when Y= 0.3. In non-prismatic beams of constant depth, the mid-span stresses were 2.15 times those in a prismatic beam for β=4.0 and Y = 0.5; for β =2.0 the stresses were 1.45 times when y = 0.5 but only 1.22 times when Y = 0.3.

Two-span continuous rectangular beam

The thermal response of two-span continuous beam will be different from that of multi-span beams. The increase in the section is usually confined to the intermediate support region, Figure 1. In this case d_2 and b_2 refer to the depth and thickness at the end support. The intermediate support moments can be computed as

$$m = E\, l_1 \left(\frac{r \propto}{d_2}\right) \left[\frac{l_1}{l_2}\right] \quad ...(4)$$

where,

$$l_1 = 3\, k\, k_1 [y_1^2\, k_1^2 + 2\, y \{y\, k_1 - (y\, k - k_1)\, l_n\, k\}]$$

$$l_2 = [2\, k^3\, k_1^3\, y_1^3\, x\, 3\, k \{2\, k^2\, y^3\, l_2\, k - k_1^2\, (1 - k^2\, y_1^2)\, y - 2\, k\, k_1\, y^2\, (y - y_1\, k_1)\}]$$

l_1 = moment of intertia of the support section.

The support moments in two-span beam are plotted as the ratio (m/m_1) against K for various values of Y in Figure 5(a). In this case, y varies from 0.0 (prismatic beam) to 1.0. The support moments in a two-span structure increase much more rapidly with Y and K compared to those in a multi-span structure. The increase in the support moment in two-span structures when the depth was varied linearly over the entire span was more than nine times that in a prismatic structure of depth d_2; the corresponding increase in the multi-span structure was about three times that in a prismatic structure. However, unlike that in a multi-span structure, it is the maximum section in the span, viz. the support section, that is subjected to the maximum moment. The section modulus of the support section for K= 4.0 will be 16 times that of the section of depth d2. Since the moments increase 9 times, the stresses at the support section would be (9/16) times that in a prismatic structure. Thus, the stresses in a two-span non-prismatic beam decrease compared to a prismatic structure. Similar features can be observed in a non-prismatic rectangular beam of constant depth but of varying thickness. The moment at the support can be determined as

$$m = E\, l_1 \left(\frac{r \propto}{d}\right) \left[\frac{l_1}{l_2}\right] \quad ...(5)$$

where,

$$l_1 = 3\, \beta_1^3$$

$$l_2 = \beta [2\, y_1\, \beta_1^3 . 3\, y \{(\beta^2 - 1)\, y^2 - 4\, y\, \beta_1\, (y - y_2\, \beta_1) - 2\, (y - y_1\, \beta)^2\, l_n\, \beta\}]$$

The values of (m/m_1) are shown in Figure 5(b). Even in this case it can be seen that the support moments are higher when compared to a multi-span bridge for the same values of Y and f. However, the relative increase in the section modulus at the support is more than the relative increase in the moments compared to a prismatic beam with the same section as that at the end support. For instance, the support moment for Y= 1.0 and β = 4.0 is about three times and the section modulus is four times the corresponding values in prismatic beam of thickness b_2 . Thus, though the support moments are higher in the non-prismatic beam, the support section will be subjected to 3/4th

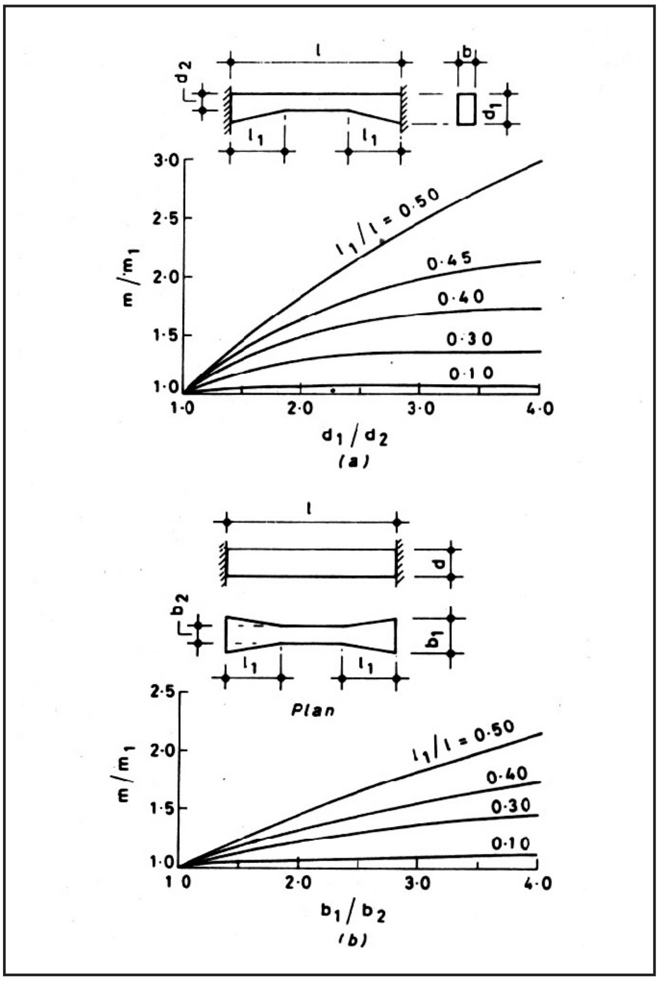

Figure 4. Variation of the restraint moment ratio (m/m_1) in rectangular beams with encaster ends for the LTD

of the stresses in a prismatic beam. Figure 5(b) also indicates that the reduction in the stresses at the support section will become more significant as Y is decreased for a given value of β, since the increase in the moments will be smaller but the support section remains the same. It may be noted that the value of the expression within the square brackets in equations 2 and 3 tends to a value equal to 1.0, while that for equations 4 and 5 tends to a value equal to 1.5 as y approaches zero, or as K or approaches unity (prismatic beam).

(To be continued)

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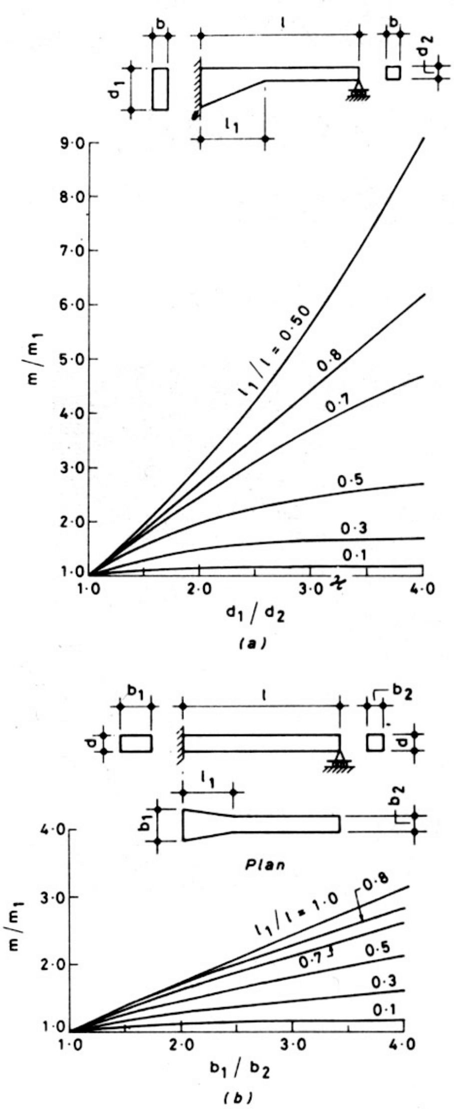

Figure 5. Variation of the support moment ratio (m/m_1) in two-span rectangular beams for the LTD