# Temperature stresses in concrete box-girder bridges

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*In the August-September 1987 issues of the Journal, the author had presented an in-depth study on critical temperature distributions in concrete bridges for Indian summer conditions. Stresses in straight and curved box girders of various depths are now computed for the temperature distributions proposed in the earlier paper. It is shown that cracking of the top slab in transverse flexure reduces the transverse bending moments in the top and bottom slabs of the box girders, and that ignoring the curvature in plan of the box girder would be on the conservative side.*

Temperature distributions for various elements of concrete bridges were proposed in an earlier paper based on the environmental data of New Delhi'. These distributions are simplified versions of the computed temperature variations in concrete elements of various thicknesses based on linear heat flow theory. It was shown that the soffit tensile stresses obtained for bridges of various configurations under the computed and proposed temparature distributions agreed adequately for several cases considered. This paper presents further results of a study on single-cell box girder bridges. The influence of cracking of the structure in transverse flexure and that of the curvature of the bridge in plan are discussed in this paper. Only two-span continuous structures are considered in this study, as it is the most severe case for longitudinal temperature stresses.

## Temperature distributions

The temperature distributions proposed for the Indian summer conditions are summarised in Figures 1 and 2. The temperature distributions in Figure 1 are for determining the longitudinal stresses in a bridge structure. Various possible structural elements are taken into account. In the case of a box girder, two cases of air-cell (the free space between top and bottom slabs and the webs) are considered, namely, ventilated (the temperature inside the air-cell at the ambient temperature) and unventilated or closed air-cell. Though it is not strictly, correct, the temperature distribution proposed for the slabs with both the sides exposed can be used for the slab above .a ventilated air-cell. The temperatures for intermediate values of thicknesses may be interpolated for the slabs, where uniform temperatures are proposed. The temperature distributions of Figure 2 are to be considered for transverse bending moments, particularly in box giders.

## Analysis

The thermo-elastic analysis of the structure was done by the finite strip method for both straight bridges and the bridges curved in plan[2]. The cross-section considered is shown in Figure 3. Only half the cross-section was analysed for straight bridges, taking advantage of the symmetry about the central vertical plane. However, for curved bridges the entire section has to be analysed. The length of the bridges was assumed to be 50m, though it is immaterial for thermal analysis of straight bridges. The effects of the intermediate diaphragms were not considered in the analysis and the intermediate supports were assumed to be point supports under the webs with no lateral restraint. These assumptions simplify the analysis considerably, but do not influence the longitudinal stresses significantly because of the symmetry of the structure and of the thermal loading about the central support. However, the transverse moments will be affected in the vicinity of the intermediate diaphragms, if any. This effect depends upon the restraint offered by the diaphragms and their spacing. Nevertheless, the effect is local and negligible at longer distances from the diaphragms.

In the analyses reported in this paper, the coefficient of thermal expansion of concrete was taken as 10.8 x $10^{-6}$ m/m/K, the Young's modulus of elasticity as 35GN/$m^2$ and the Poisson's ratio as 0.15. The cross-section of the straight bridges was divided into 18 strips and that of the curved bridges into 34 strips (50 terms of the series were adequate for convergence).

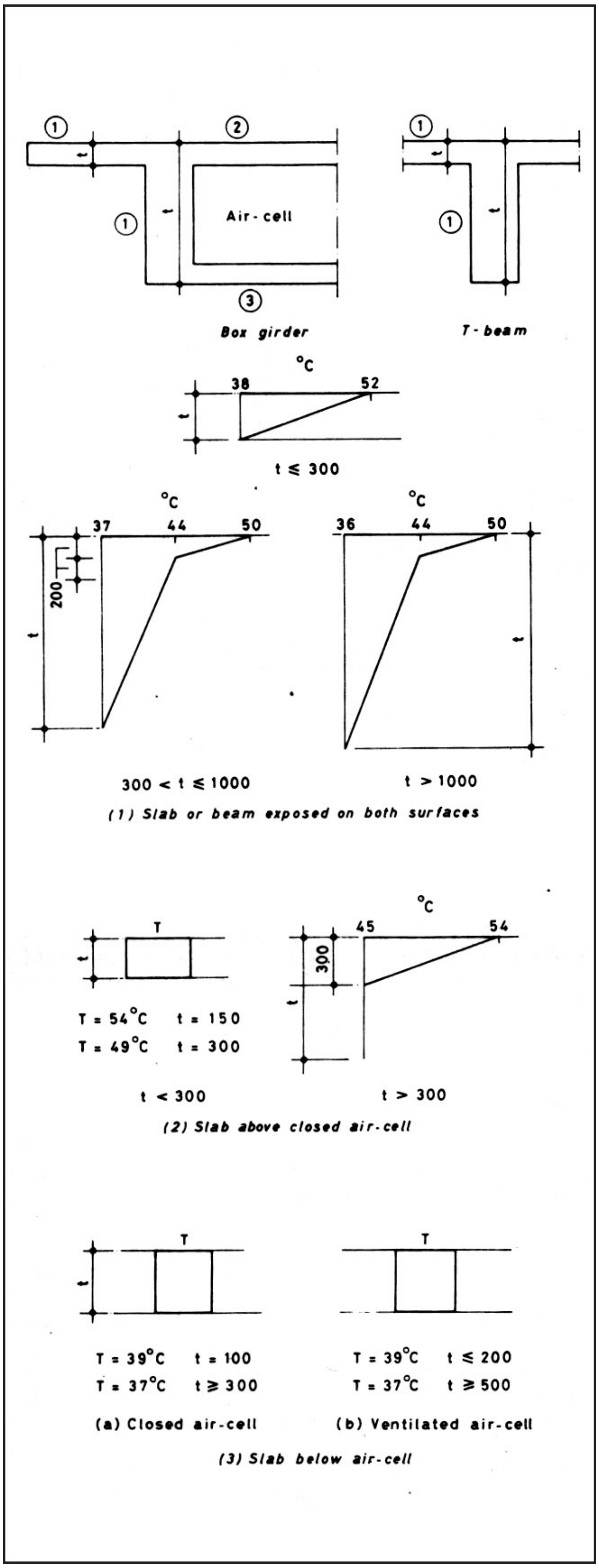

**Figure 1. Proposed temperature distribution for longitudinal stresses (T = temperature, t = thickness)**

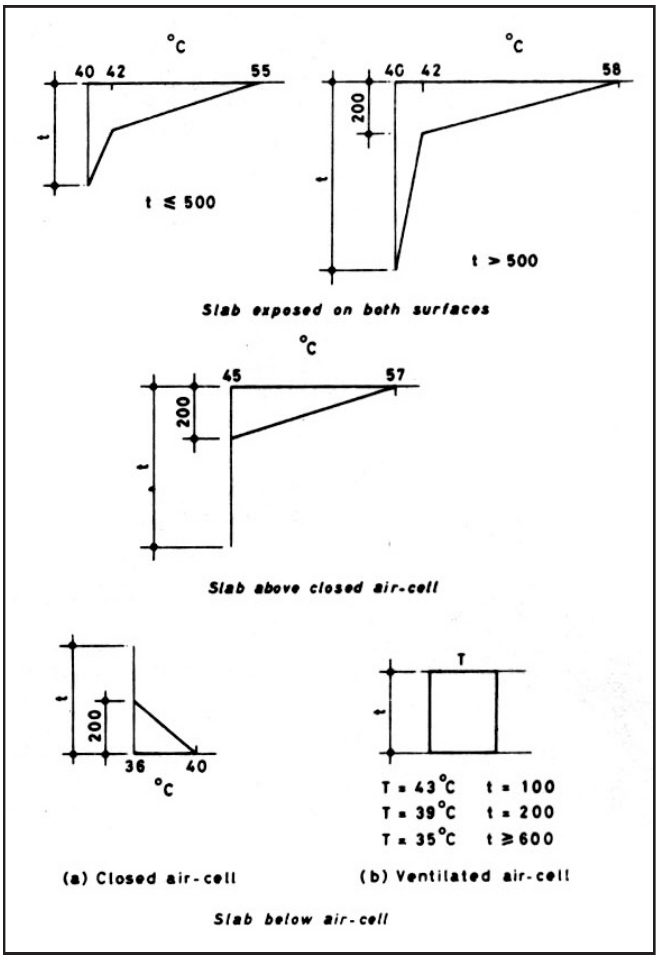

**Figure 2. Proposed temperature distribution for transverse moments (T = temperature, t = thickness)**

The temperature loads were transformed into equivalent static laods as[3],

$$[b_m] = [B_m] [T] [D] [x] dv$$

where,

$[b]$ = load matrix
$[B]$ = strain-displacement matrix
$[T]$ = temperature matrix
$[D]$ = rigidity matrix
$[x]$ = thermal coefficient of expansion matrix
$m$ = number of the term.

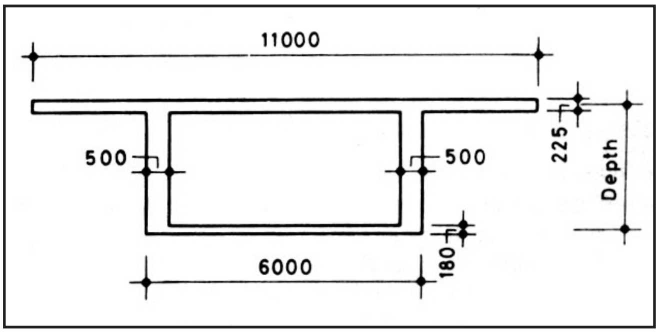

**Figure 3. Cross-section of the box girder assumed in the analyses**

The soffit tensile stresses were computed at a distance of 0.5m from the intermediate support. The transverse moments will be the same at any section sufficiently away from the supports. The central supports were simulated by means of the flexibility approach[2] . It is also assumed that the design temperature distributions developed for the straight bridges are valid for curved bridges.

## Influence of cracking

The problem of cracking in reinforced and partially-prestressed concrete bridges in longitudinal flexure was dealt by Thurston and Cooke and others in considerable detail[4, 5]. In the case of reinforced concrete girders, the displacements under thermal loading can be computed for the uncracked sections as a good approximation and the stresses can be computed by taking cracking into account. The procedure is much more involved for the thermal analysis of partially-prestressed concrete girders, requiring an estimate of the extent of the cracking depth. Thurston computed the influence of cracking on transverse moments and concluded that the transverse bending moments were reduced by about 90 percent on account of cracking in reinforced concrete slabs[4]. However, the slabs considered by Thurston were only lightly reinforced and the moment of inertia in the cracked state was reduced to about 10 percent of its value in the uncracked state.

The cracking of a slab generally reduces its flexural rigidity to about one-third to one-fifth of its value in the uncracked state, depending on the slab thickness, the extent of cracking and the amount of reinforcement. In the present study, the box girder is assumed to have cracked in transverse flexure only and the in-plane stiffness and the longitudinal flexural rigidity are assumed to be unaffected. The ratio of the longitudinal to transverse flexural rigidities ($D_y/D_x$ ) of the strips was varied from 1.0 to 5.0. The resulting longitudinal soffit tensile stresses at the intermediate support ($d_t$) and the transverse moments ($M_t$) are plotted in Figures 4 and 5. The depth of the box girders was varied from 1.0m to 4.0m. The transverse moments were considered to be positive when they caused tension on the inner side of the air-cell; i.e., sagging moments in the top slab, and hogging moments in the bottom slab were positive.

The transverse moments in the slabs were found to decrease significantly as the ratio $D_y/D_x$, was increased from 1.0 to 5.0, the reduction being greater initially. There was a reduction of about 50 percent in Mt as the ratio $D_y/D_x$, was increased from 1.0 to 2.0. The values of $M_t$ for $D_y/D_x = 5.0$ were about a quarter of the values in uncracked sections and, thus, may still be significant for the design of the top slab. Similar tendencies were noticed for the box girders with ventilated air-cell, but the values were not shown in Figure 4 as their variation was not very different.

The values of at on the other hand appeared to be little affected by the cracking in transverse flexure as can be noticed in Figure 5. There was a slight reduction in a t as the ratio $D_y/D_x$, was increased from 1.0 to 2.0, the maximum reduction being about 15 percent for the 1.0-m deep box girder with ventilated air-cell; but thereafter the values of $0_t$ remained almost constant.

It can be further noticed from Figures 4 and 5 that the longitudinal stresses were reduced as the depth of the girder was increased. The-transverse moments in the top slab increased and those in the bottom slab decreased as the depth of the box girder was increased.

## Influence of curvature

The salient features of two-span curved bridges considered in the analyses are indicated in Figure 6. The length of the box girder along the central axis was taken to be 50m and the included angle Ø was varied from 0.0005 to 1.0 radian in order

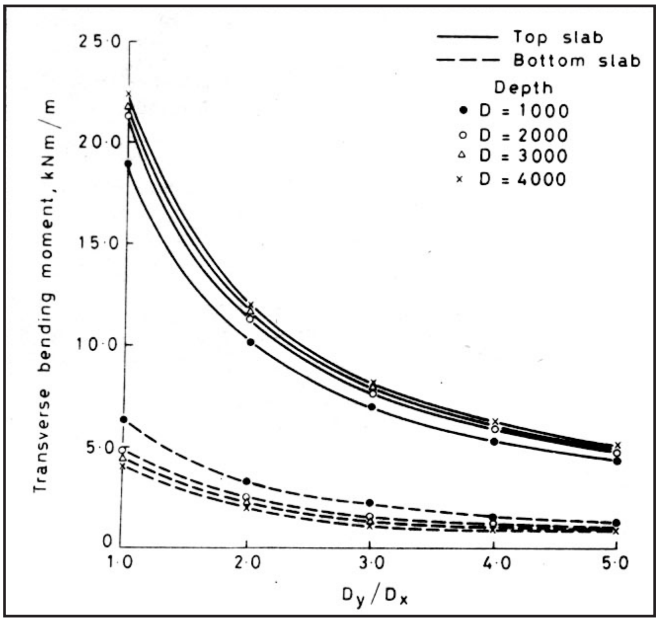

**Figure 4. Variation of transverse moments in box girders with $D_y/D_x$**

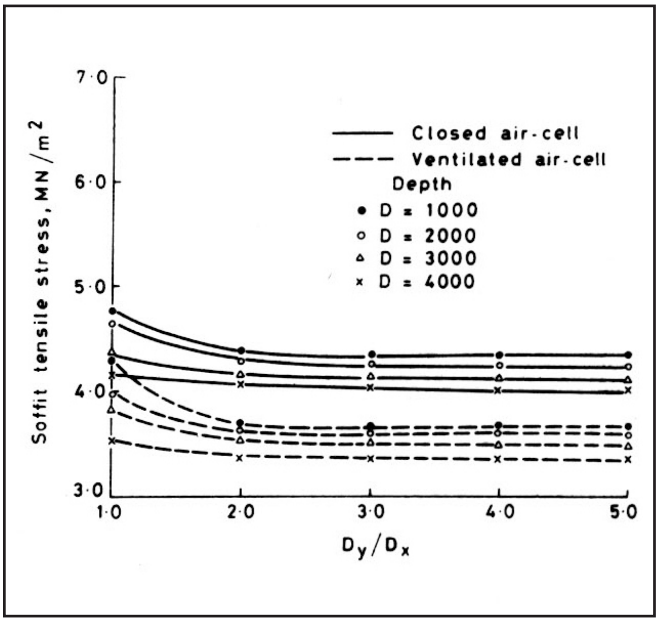

**Figure 5. Vadalion of soffit tensile stresses in two-span continuous box girders with $D_y/D_x$**

to obtain various curvatures in plan. The central supports were assumed to be on the radial line bisecting the included angle Ø.

## Longitudinal stresses

The longitudinal stresses $\sigma_t$ are plotted in Figure 7 for the inner and outer webs. The values of $0_t$ will be the same for inner and outer webs for very small angles of Ø, but will differ significantly as Ø is increased. The stresses at the outer webs tended to be more than those at the inner webs for depths of the box girder 2.0m or less, but the trend was reversed for larger depths. The outer web of a curved girder is longer than the inner web and thus deflects more in the vertical direction. Box girders of smaller depths are relatively more rigid in transverse flexure than those of larger depths[6] .

Thus, the bearing reaction tends to be larger under the inner webs than that under the outer webs at the central supports for box girders of smaller depths, thereby increasing $\sigma_t$ under the inner webs. For box girders of larger depths, which are relatively more flexible in transverse flexure, the bearing reaction is more under the outer webs, which in turn increases $0_t$ at the outer webs.

It can also be noticed in Figure 7 that a, decreases with an increase in the included angle. $\sigma_t$ except in the case of a box girder of 1.0-m depth. Even in the case of a 1.0-m deep box girder, the mean a, of the webs decreases as $\sigma_t$ is increased. This reduction in stresses is because of the reduction in chord distance between the supports as $\sigma_t$ is increased, Figure 6 Salient features of the curved box girders analysed which reduces the mean bearing reaction at the central supports. The reduction in at is more significantin box girders of larger depths. It will thus be conservative if the influence of curvature in plan on at is neglected in the thermal analysis for box girders of depths greater than 1.0m.

## Transverse moments

Transverse moments in top and bottom slabs of box girders are plotted in Figure 8 for various values of . It was found during the analysis that the transverse moments increased slightly at the inner webs and decreased at the outer webs as Ø was increased. The mean values of the moments, however, remained the same. The values plotted in Figure 8 pertain to the inner webs and are larger than those at the outer webs. The increase in the moments is not very significant, being about 13 percent in 4.0-m deep box girder and less than 7 percent in 1.0-m deep box girder as 4) is increased from 0 to 1.0 radian. A curved slab, when heated, deflects in its plane more towards the centre. The top slab of the box girder being at a higher temperature than the lower slab, the girder deflects towards the centre more at the top than at the bottom. This causes higher $M_t$ at the inner web than at the outer web.

However, the positive $M_t$ is critical in the region midway between the webs. In this region $M_t$ remains the same regardless of Ø. Thus, it would appear that the influence of curvature in plan of the box girder on $M_t$ can be neglected.

## Conclusions

Transverse moment due to insolation effects in the top slab of a box girder increases as the girder depth is increased, while that in the bottom slab decreases. The maximum soffit tensile stress in the two-span bridge decreases as the girder depth is increased for the cross-section investigated in this paper.

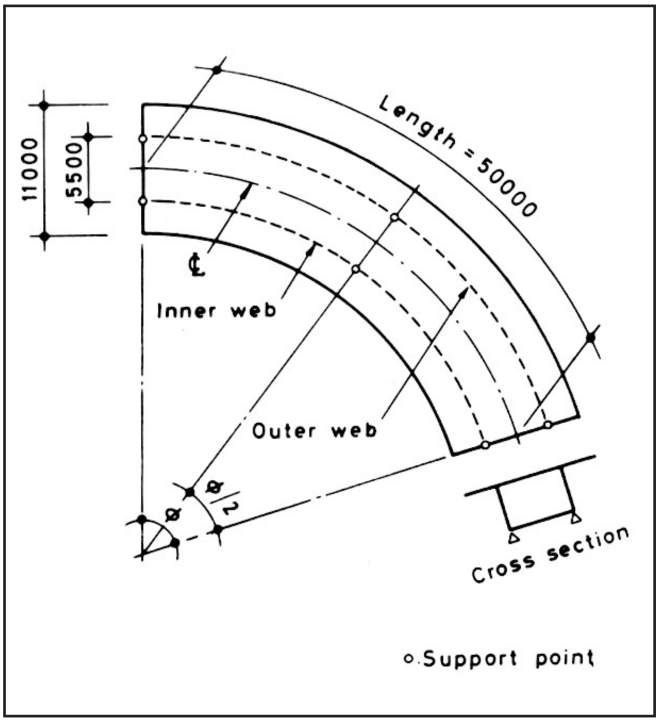

**Figure 6. Salient features of the curved box girders analysed**

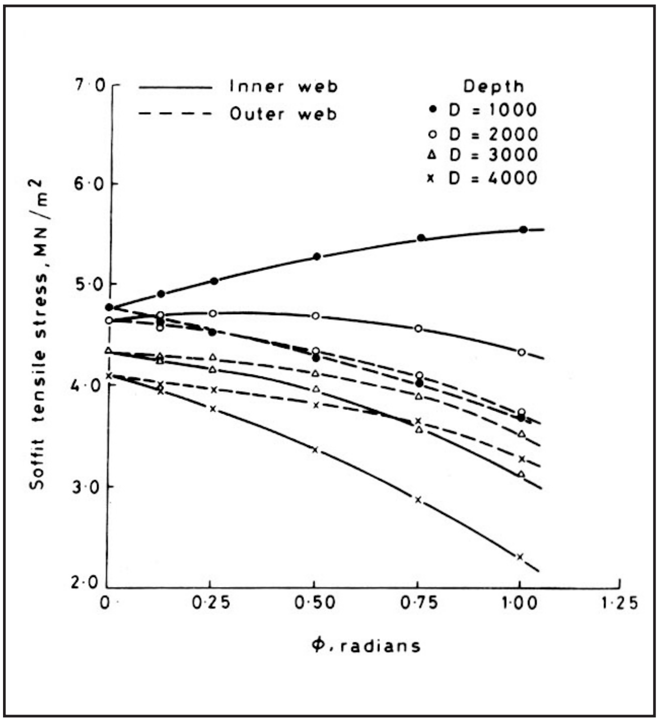

**Figure 7. Soffit tensile stresses in 50-m long, curved, two-span continuous box girders**

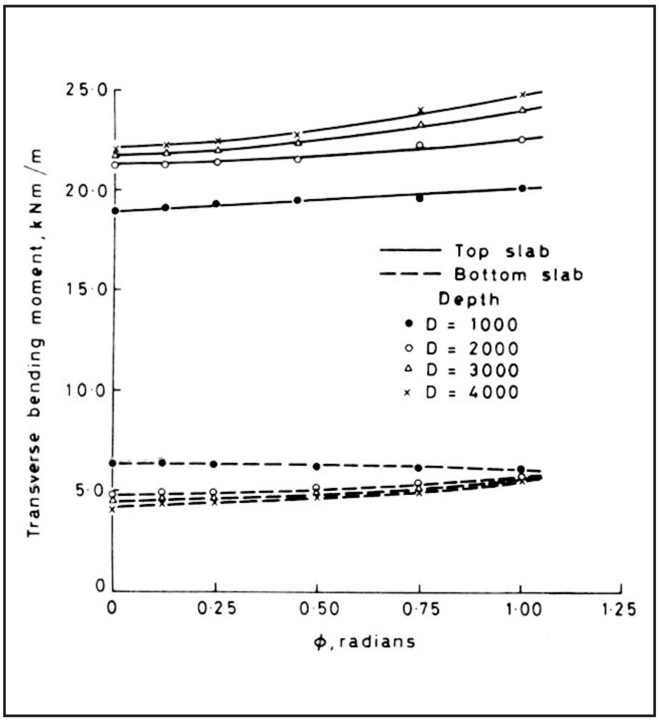

**Figure 8. Transverse moments in curved box girders**

Cracking of the girder in transverse flexure reduces the transverse bending moments significantly, depending upon the extent of cracking and the reinforcement. The moments may still be significant in the cracked slab for the temperature distributions assumed in this paper. The longitudinal stresses, however, are orify slightly reduced as a result of transverse flexural cracking, it being assumed that the inplane stiffness of the girder remains unchanged.

The longitudinal soffit stresses decrease as the curvature in plan of the girder is increased, except for girders of depths of about 1.0m. The reduction in soffit tensile stresses is more significant for girders df ltirger depths. Nevertheless, it would be on the conservative side if the curvature in plan of the girder is neglected in thermal computations, except for girders of small depths.

The transverse moments are increased slightly as the curvature in plan of the girder is increased.

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