Influence of box shape on structural behaviour of RC box girder bridges

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The present study is concerned with the analytical investigations on influence of box cross-sectional shape on the structural behaviour of reinforced concrete (RC) box girder bridges. In this paper the structural behaviour of box girder bridges has been studied in terms of deflection, longitudinal and transverse bending stresses, and the shear lag coefficient. To this end two box shapes namely, rectangular and trapezoidal have been considered for the aforementioned study. ANSYS, a general purpose finite element analysis software, has been used as the basic platform for modelling and carrying out the three dimensional linear elastic finite element analysis of box girder bridges considered in this study. The outcome of this study has shown that as a whole the structural response of bridge with rectangular box section is better than trapezoidal box section.

Keywords: *Box girder bridge, structural behaviour, shear lag, finite element analysis, patch load, ANSYS.*

Reinforced concrete (RC) box girder bridges are generally considerable to have better stability, serviceability, economy, aesthetic appearance and structural efficiency due to its high torsional stiffness. A box girder consists of top slab, bottom slab and webs. The webs of the box girder are generally inclined as a means of enhancing the appearance of the box girder bridge. Due to inclination of webs, the transverse span of the bottom slab reduces but it increases for top slab in comparison to box girder with vertical webs. The provision of inclined webs not only affects the transverse stresses but it also affects the longitudinal stress distribution in top and bottom slabs. In this paper an attempt has been made to investigate the influence of box shape on the structural behaviour of the box girder bridge.

Structural behaviour

The structural behaviour of box girder bridges is quite complex due to its three-dimensional geometry. Under the self-weight as well as for symmetrical loadings, the box section primarily experiences longitudinal and transverse bending. However, in the presence of unsymmetrical loading, the box sections are subjected to torsion in addition to bending in longitudinal and transverse directions and these three actions are generally referred to as simple beam actions.

Moreover, since the walls of the box girder are made thin in order to reduce the self-weight of the bridge, distortion and warping are also developed in the box cross-section in addition to simple beam actions. Distortion is defined as the alteration of the cross-section due to transverse bending and warping is defined as out of plane displacement of cross-sectional fibres in longitudinal direction. Distortion may occur in symmetric as well as anti-symmetric (or torsional) loading while warping arises due to anti symmetrical loading only. Warping and distortion in the box sections occur due to absence or insufficient number of rigid diaphragms.

Thus, the structural response of the box girder bridge is a combination of five structural actions namely longitudinal bending, transverse bending, torsion, distortion and warping.

Furthermore, in box sections the longitudinal bending stresses in top/bottom slabs in the regions close to the webs are found higher in comparison to other points on the top/bottom slab at that section on respective slabs. The phenomenon associated with this non-uniform longitudinal stress distribution is called shear lag[1]. Simple bending theory is unable to reflect the shear lag phenomenon. Shear lag is quantified in the form of nondimensional parameter known as coefficient of shear lag (CSL). CSL at any point in the top/bottom slab is defined as the ratio of actual longitudinal bending stress in the top/bottom slab under the given loading condition to the longitudinal

Table 1. Geometrical properties of the sections

Shape	Cross-sectional area, m^2	Moment of inertia, m^4	Position of neutral axis from top fibre, m
Rectangular	5.61	5.84	1.023
Trapezoidal	5.31	5.45	0.982

bending stress calculated at that point on the basis of simple bending theory under the same loading condition. CSL is found greater than one near the web and top/bottom slab junctions while it is found less than one away from web and top/bottom slab junctions. In wide flanged sections, CSL may reach up to 1.3 near the web flange junction[2]. Simple bending theory in conjunction with CSL may be used to determine the actual longitudinal stresses including shear lag effect. To determine the actual longitudinal bending stress, the longitudinal bending stresses calculated on the basis of simple bending theory are multiplied by CSL to incorporate shear lag effect. In this paper, a comparative study has been made for deflection, longitudinal and transverse bending stresses and coefficient of shear lag for rectangular and trapezoidal sections.

Method of analysis

For the preliminary design consideration of box girder bridges, one-dimensional analysis based on the simple bending theory is generally used by the designers to predict the longitudinal stresses and deflection.

However, for the design of box girder bridges simple bending theory is not sufficient as it is unable to account for the transverse bending in the section. To overcome this problem, other analytical methods based on orthotropic plate theory, grillage theory and beam on elastic foundation analogy were used for the analysis of box girder bridges[3]. These methods of analysis were incapable of reflecting the shear lag effect and, therefore, Reissner used the energy method to solve the shear lag problem[4].

The aforementioned methods of analysis are nevertheless limited in scope and in general do not apply to arbitrary shapes and loading conditions. With the advent of modern tools of analysis such as finite element method, it is now possible to predict the complex behaviour of box girder including the shear lag effect. Moffat and Dowling used finite element method to study the shear lag effect in box girders5. The finite element approach is gaining a lot of favour as it can conveniently be extended to non-linear analysis to study the post-cracking behaviour of the bridges.

Cracking in the box girder may initiate even under self weight of the bridge. However, for the simplified design of bridge structures, the bridges may be assumed un-cracked up to service load conditions and consequently its behaviour may be assumed linear elastic up to the service loads. The bridges considered in this study have been analysed for dead load and service load conditions on uncracked basis using the linear elastic finite element analysis. The finite element analysis of the bridges has been performed using the general purpose finite element software ANSYS[6].

Geometric configurations

To study the structural behaviour of box girder bridge, a two-lane simply supported single cell RC box girder bridge of span 30 m is considered. As per the general construction practice, the diaphragms are assumed at the supports only. In order to study the influence of box-shape on structural behaviour of box girder bridge, the bridge has been analysed with two cross-sectional shapes namely rectangular and trapezoidal. Since the present study intends to investigate the effect of shape only, the top slab width, overall depth and thickness of all box elements namely top slab, bottom slab and webs has been kept constant.

The top slab width and overall depth of the sections have been taken as 9.6 m and 2.6 m, respectively. The thickness of top slab, bottom slab and webs is taken as 300 mm for both the sections and the wearing coat thickness is taken as 75 mm. The geometrical details of rectangular and trapezoidal sections considered in this study are shown in Figure 1 and the geometrical properties are shown in Table 1.

Loading

The bridges considered in this study have been analysed for dead load (DL) and service load (SL) conditions. For dead load calculation, the density of concrete and wearing coat material has been taken as 25 kN/m^3 and 22 kN/m^3, respectively. For live load (LL), the Indian Road Congress specifications have recommended various types of standard loadings[7]. Type of loading to be used depends on its use, type of traffic and location of the bridge. Normally, two lane bridges on national

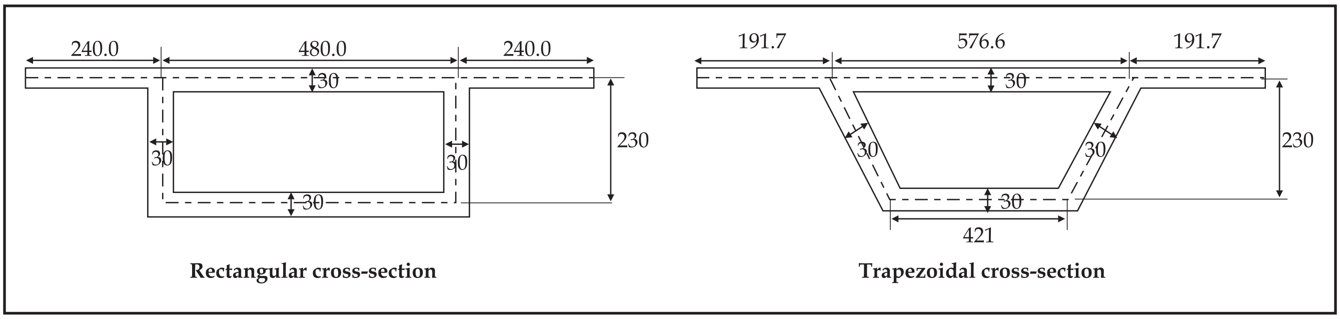


Figure 1. Geometrical details of the sections

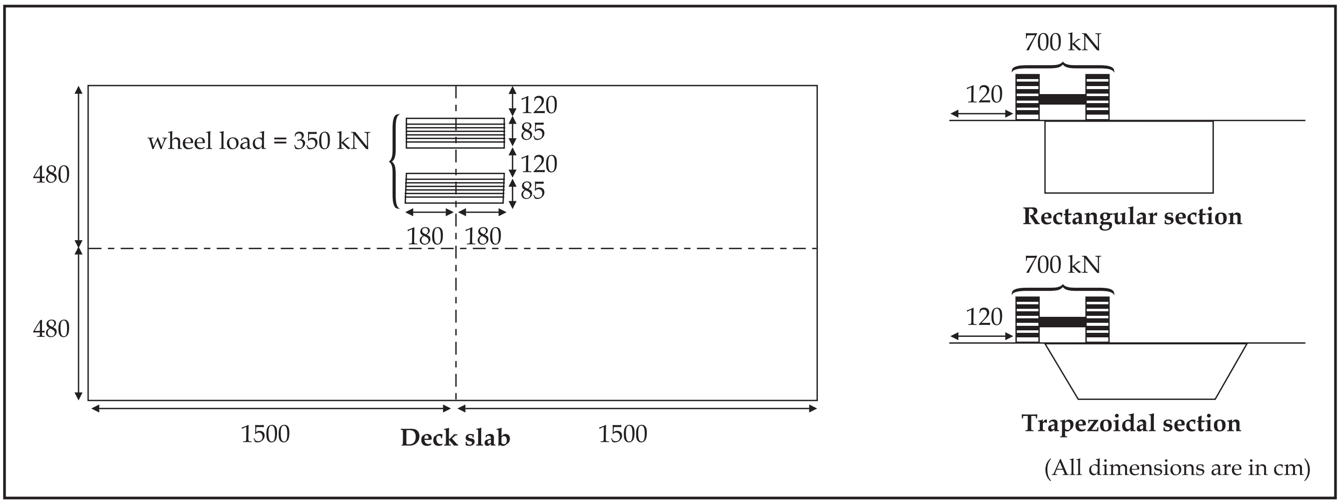


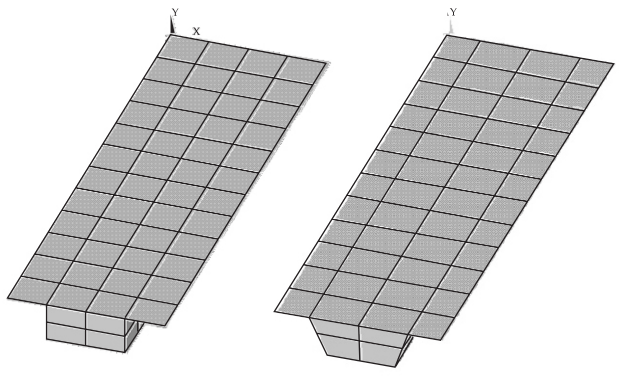
Figure 2. Position of live load on bridge

highways and state highways are designed for IRC class AA loading on one lane and checked for class A loading on both the lanes. IRC class AA loading consists of a 700 kN tracked vehicle or a wheeled vehicle of total load of 400 kN while the IRC class A loading consists of a train of wheels with a total load of 554 kN. Live loads when moving on the bridge can occupy any possible position on the deck slab.

However, it is required from design point of view to keep the live loads on deck slab in such a way that it results in the maximum moments and shears. In this limited study, only class AA tracked loading has been considered and has been placed at mid-span to develop maximum longitudinal moment and at minimum clearance from the inner edge of the kerb to produce maximum torsion in the section. Details like clearance, wheel contact areas, load intensity and spacing of wheels are shown in Figure 2.

Finite element modelling

The generic analysis of box girder bridges is high complicated due to thin walls. This has led to the reliance on the finite element method using various shell elements[8, 9]. ANSYS, the most commonly used software in structural engineering was selected as the basic platform for modelling and carrying out the analysis of box girder bridges.



Rectangular section
Trapezoidal section

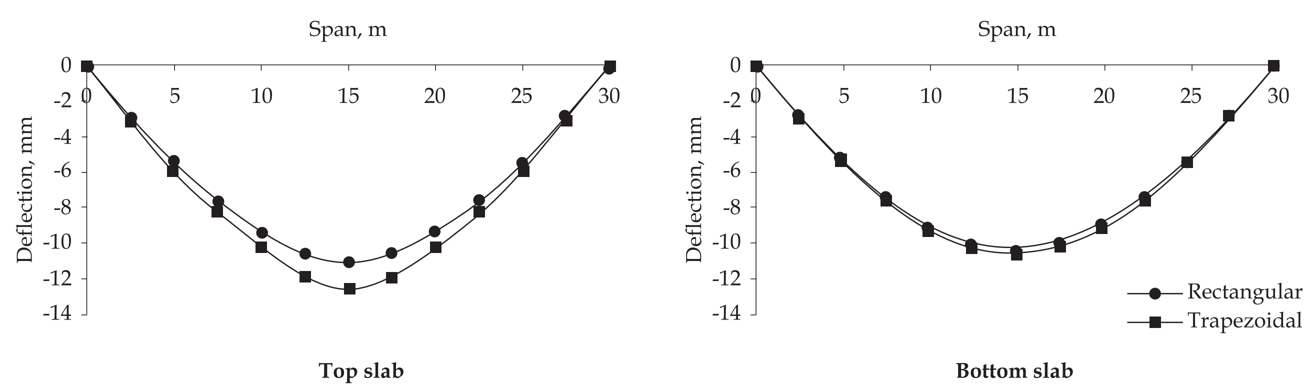
Figure 3. Discretisation of box girder bridges used for ANSYS analysis

The bridges considered in this study were modelled using Shell91 element available in ANSYS element library. Shell91 element has been adopted because of its capability to do layered finite element analysis. The top slab was modelled with two layered Shell91 elements (one for concrete and other for wearing coat) while the bottom slab and webs were modelled as single layer Shell91 elements. Shell91 is a 8-noded element having 6 degrees of freedom at each node. Three dimensional finite element models of bridge with rectangular and trapezoidal sections considered in this study are shown in Figure 3. For the finite element analysis, the modulus of elasticity and Poisson's ratio for concrete along with wearing coat have been taken as 3.61×107 kN/m^2 and 0.15, respectively.

The wheel loads of IRC class AA tracked vehicle have a finite area and cannot be treated as point loads and therefore, these loads have been considered as patch loads wherein the load is assumed to be uniformly distributed over the small contact

Table 2. Comparison of maximum deflection and maximum longitudinal and maximum transverse stresses

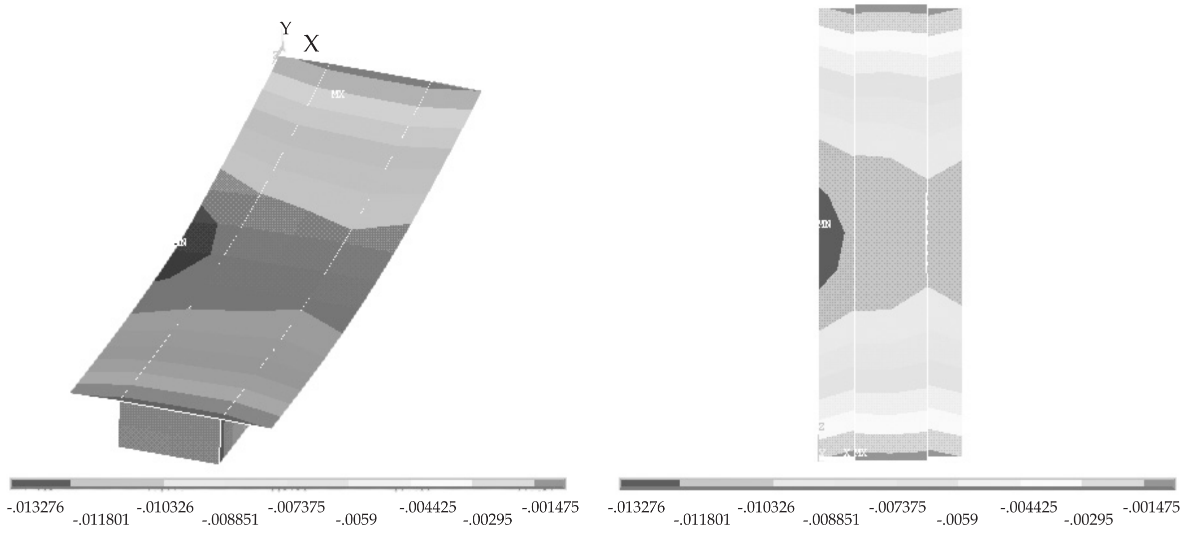
Parameter	Top slab			Bottom slab		
	Rectangular section	Trapezoidal section	Change, %	Rectangular section	Trapezoidal section	Change, %
Max. deflection, mm	11.07	12.54	13.30	11.00	11.21	1.90
Max. transverse stress, kN/m^2	3661	4771	30.30	3964	4587	15.72
Max. longitudinal stress, kN/m^2	5048	5298	4.96	6857	7105	3.62



Top slab

Bottom slab

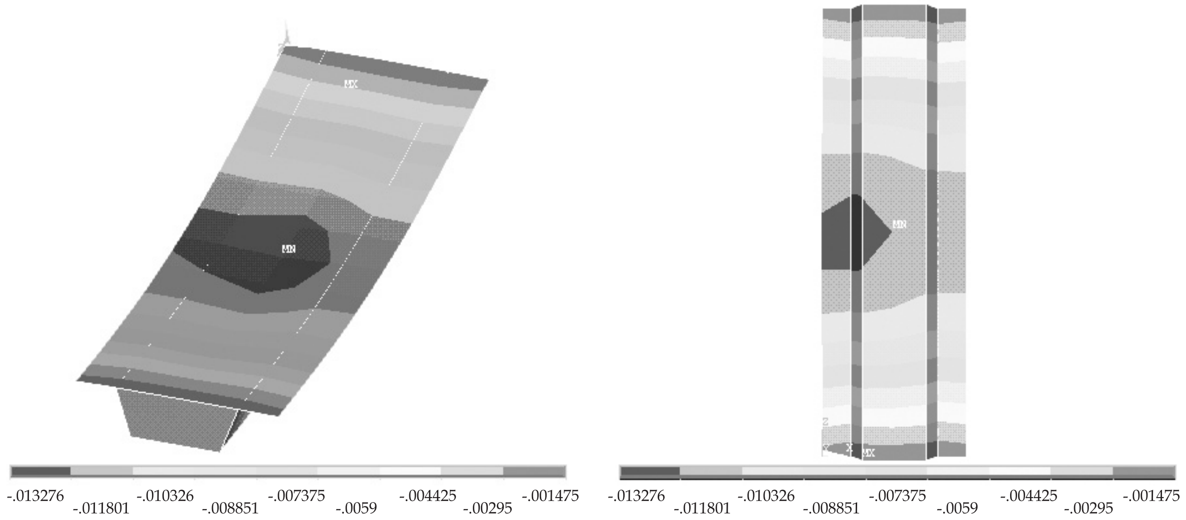
Figure 4. Deflection profiles of top and bottom slabs



Top slab

Bottom slab

(a) Rectangular section

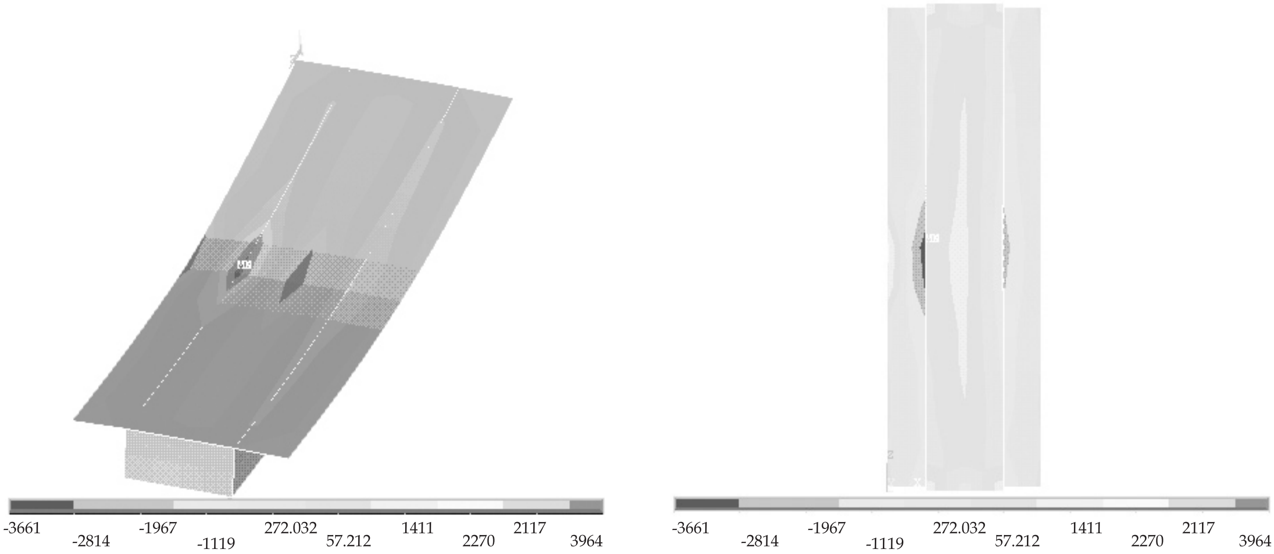


Top slab

Bottom slab

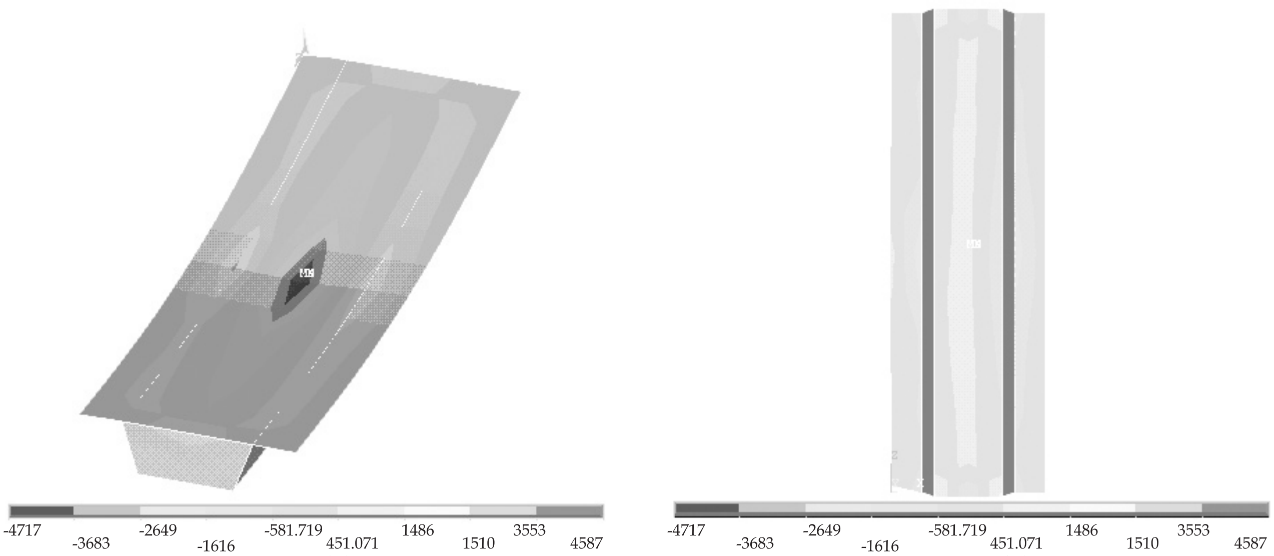
(b) Trapezoidal section

Figure 5. Influence of box shape on deflection pattern



Top slab Bottom slab

(a) Rectangular section



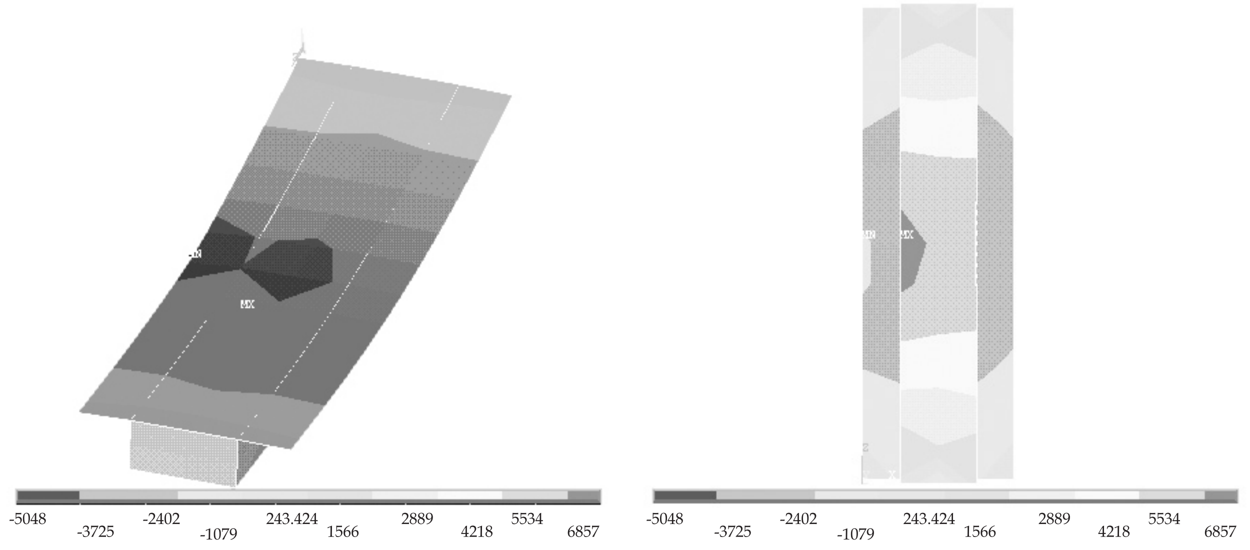
Top slab Bottom slab

(b) Trapezoidal section

Figure 6. Influence of box shape on transverse bending stress at service load condition

area between wheel and deck surface as shown in Figure 2. For the finite element analysis, the geometry of the patch is treated as sub-element similar to the parent element used to model the deck slab. The equivalent normal loads at nodes of the sub-element due to the uniformly distributed patch load are calculated using the shape functions of element. The position of sub-element nodes in Cartesian coordinate system is known from geometry. However, in order to transfer the sub-element nodal loads to parent element nodes there is a need to determine the location of sub-element nodes in curvilinear coordinate system corresponding to parent element.

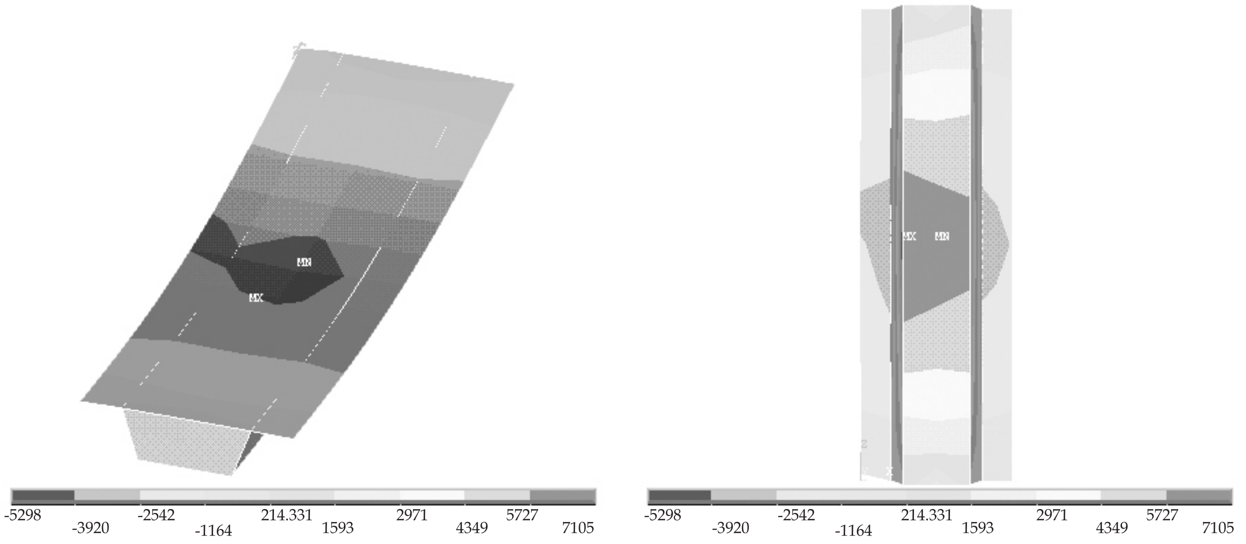
The Cartesian coordinates of sub-element nodes are converted to curvilinear coordinates corresponding to parent element using the inverse mapping technique developed by Elwi and Hrudey[10]. In the inverse mapping technique, the curvilinear coordinates of a point corresponding to its Cartesian coordinates are determined using the iterative numerical method. After knowing the sub-element nodal loads and its position in curvilinear coordinate system associated with



Top slab

Bottom slab

(a) Rectangular section



Top slab

Bottom slab

(b) Trapezoidal section

Figure 7. Influence of box shape on longitudinal bending stress at service load condition

parent element, the sub-element nodal loads are transferred to the parent element nodal nodes.

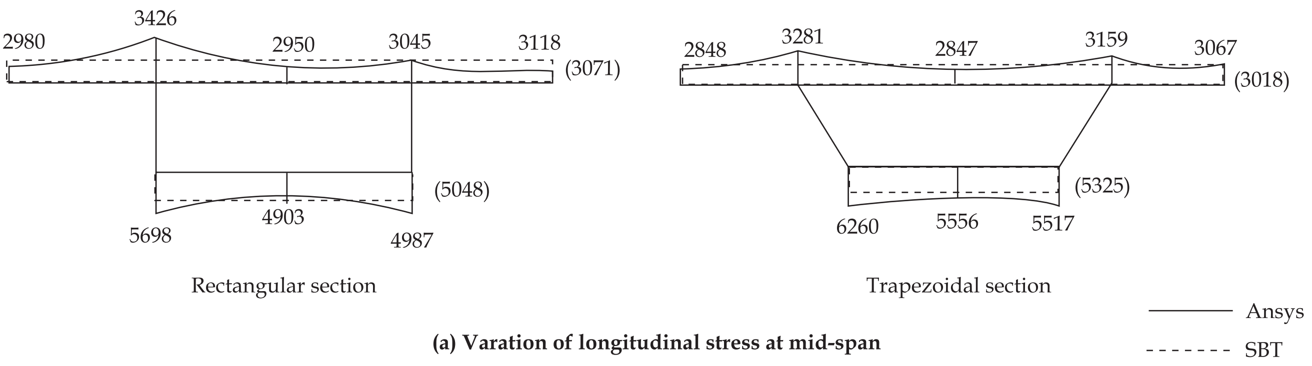
Results and observations

The aim of this study was to investigate the influence of box shape on the structural behaviour of box girder. The current study examines four structural parameters namely deflection, longitudinal bending stress, transverse bending stress and the shear lag. The analysis of box girder was carried out for dead load (DL) and service load (DL+LL) conditions.

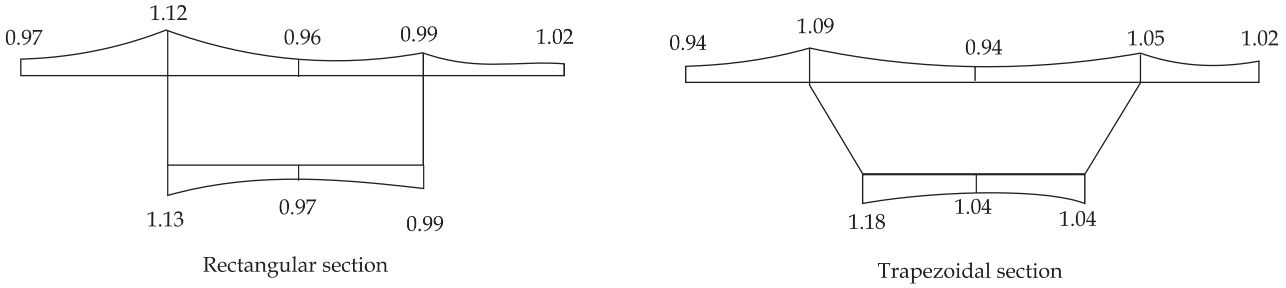
However, for brevity, ony the results of the latter case have been presented and discussed. To investigate the influence of geometry of box section on deflection, transverse stress, longitudinal stress and coefficient of shear lag, the results for these parameters are presented in Figures 4 to 8 and the magnitudes of maximum deflection, maximum transverse and maximum longitudinal stresses are summarised in Table 2. The observations from the results for deflection, transverse stress, longitudinal stress and shear lag are discussed below.

Deflection

The deflection profiles of top slab and bottom slab along the span at service load condition as obtained by finite element analysis have been plotted in Figure 4. From the figure it may be observed that for top slab the deflection profile is significantly affected by the cross-sectional shape of the box and from Table 2, it may be observed that at service load



(a) Varation of longitudinal stress at mid-span



(b) Variation of coefficient of shear lag at mid-span

Figure 8. Influence of box shape on longitudinal stress and coefficient of shear lag

conditions, maximum deflection for trapezoidal section is approximately 13% higher as compared to rectangular section.

However, this difference considerably decreases to approximately 2% in case of bottom slab. The overall deformed shapes of rectangular and trapezoidal box girder bridges at service load condition are shown in Figure 5. The figure indicates that the deflection under live load for trapezoidal section is more as compared to rectangular section thus reflecting higher torsional rigidity of rectangular section than the trapezoidal section.

Transverse stress

The three dimensional view of the bridges showing the variations in transverse bending stress in top and bottom slabs under service load condition are shown in Figure 6. From the figure it appears that in case of rectangular section higher stresses are developed in the vicinity of live load. However, in case of trapezoidal section, the high stress zone is spread over a large portion of top slab as well as for bottom slab. Moreover, it is evident from Table 2 that the maximum transverse bending stresses in trapezoidal section are approximately 30% and 16% higher in top and bottom slabs, respectively with respect to rectangular section.

Longitudinal stress

Figure 7 shows the analytical predictions for longitudinal stresses in top and bottom slabs of rectangular and trapezoidal sections under service load conditions. Since the live load has been placed at mid span at maximum permissible eccentricity in transverse direction, the maximum longitudinal bending stress in the top as well as in bottom slab occurs at mid span in the vicinity of live load for both the sections. Moreover, the figure shows that in the bottom slab of trapezoidal section the higher stress zone is spread over a large area as compared to rectangular section. Furthermore, as observed from Table 2, maximum longitudinal stresses are not significantly influenced by the shape of the section. For the trapezoidal section, maximum longitudinal stresses were observed to be approximately 5% and 4% higher in top and bottom slabs, respectively as compared to rectangular section.

Shear lag

The variation in longitudinal stress in transverse direction at mid-span obtained from Simple Bending Theory (SBT) and ANSYS have been plotted in Figure 8(a). The longitudinal stresses from SBT shown in the figure are calculated at the middle fibre of the slabs. However, the ANSYS results shown are the average of top and bottom fibre longitudinal stresses. The ratio of longitudinal stress known as coefficient of shear lag (CSL) calculated by ANSYS and SBT, is plotted in Figure 8(b). It can be observed from the figure that CSL is greater than one near the junction of top/bottom slab and web nearer to wheel load which indicates that the simple bending theory underestimates the longitudinal bending stress at these locations for both the sections.

Conversely, at mid of the top/bottom slab, the value of CSL is less than one which indicates that the simple bending theory overestimates the longitudinal stress at mid of slab. Furthermore, Figure 8(b) indicates that at the junction of top slab and web nearer to live load, the shear lag is little higher in rectangular section while in case of bottom slab, the trapezoidal section is more susceptible for shear lag effect.

Conclusion

In this paper a linear finite element analysis of the box girder bridge was carried out using the software ANSYS to investigate the influence of box shape on deflection, longitudinal and transverse bending stresses, and shear lag. The study revealed that for bottom slab, the deflection profile along the span is not significantly affected by the cross-sectional shape of the box.

However, the maximum deflection of top slab for trapezoidal section was observed approximately 13% higher as compared to the rectangular section at service load conditions. Moreover, the transverse bending stresses were found to be highly influenced by the shape of the section as compared to longitudinal stresses.

The transverse bending stresses in trapezoidal section were found to be 30% and 16% higher in top and bottom slabs, respectively. But the longitudinal stresses were observed to be merely 5% and 4 % higher in top and bottom slabs, respectively as compared to rectangular section. Additionally, the study has shown that in top slab, the shear lag effect (at web closer to live load) is more predominant in case of rectangular section as compared to trapezoidal section. Conversely, in bottom slab, the shear lag effect was found to be more for trapezoidal section as compared to rectangular section. As a whole it has been observed that the overall structural response of the rectangular section is better than the trapezoidal section.

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