# Considerations in seismic design of bridges

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This paper outlines the basic philosophy of seismic design of bridges and enumerates some of the major shortfalls of Indian codes on the subject. It further highlights the need for differentiating between buildings and bridges with regard to earthquake force levels due to the difference in the characteristics of the two types of structures. The important design features of seismic attachments of superstructure to pier / abutment caps and the state-of-art techniques of seismic protection of bridges have been discussed. Finally, design considerations relating to the emerging field of bridge retrofitting for earthquakes have been given an exposure.

#### Basic philosophy Earthquake loading

The almost universally-accepted view of the philosophy of structural design when applied to earthquake loads on a vast majority of structures including bridges can be stated as follows<sup>1</sup>.

- 1. Small to medium earthquakes should be resisted without significant damage. The stresses should, by and large, remain within elastic range. This could be viewed as a serviceability performance criteria to control damage.
- 2. Strong earthquakes should not cause collapse. This could be viewed as a safety criteria at ultimate loads.

For loads other than those due to earthquakes (for example, dead load, live load, wind load, etc.) cracking of concrete and yielding of steel in reinforced concrete would be considered as a sign of structural inadequacy. However, during strong earthquakes the loads greatly exceed those calculated by codal provisions and reliance is placed on the capacity of structures to dissipate enough energy by imparting ductility at the yielding joints so as to prevent collapse.

To arrive at the earthquake loading for structures at a given

location, it is not only necessary to fix a value of the maximum ground motion but also the frequency with which such events may be expected. The design life or period of exposure to seismic risk of a structure (often assumed at 50 to 100 years) is yet another variable which must be taken into account. The evaluation of earthquake loading has therefore to be viewed as a stochastic process.

Be that as it may, the following equation is accepted as the basis for predicting ground motion occurrence<sup>2</sup>.

$$ts = \left\{1 - (1 - p)^{\frac{1}{1}}\right\}^{-1}$$
(1)

where,

ts = return period (in years)

- t = design life or remaining design life or duration of exposure to seismic risk (in years)
- p = probability of exceedence.

For convenience, the equation has been plotted in Figure 1. From this figure the following can be deduced :

- 1. Considering a design life of 50 years for the structure and accepting only a 10 percent probability of exceedence, the return period of the earthquake would be 475 years.
- 2. Considering a design life of 100 years for the structure and accepting a 50 percent probability of exceedence, the return period of the earthquake would be 150 years.

Whereas approach (i) has been suggested for buildings, bridges and other structures in documents like ATC-3<sup>3</sup>, ATC-6<sup>4</sup>, NEHRP<sup>5</sup> and SEAOC<sup>4</sup>, approach (ii) forms the basis of recommendations of the Bridge Committee of the New Zealand National Society for Earthquake Engineering<sup>7</sup> for bridges



Figure 1. Relation between return period *ts*, design life *t*, and probability of exceedence *p* 

specifically.

An interesting comparison of how the return periods at a particular site can be correlated to the peak ground acceleration is shown in Figure  $2^8$ . For example, for a site with "moderate" seismicity, if the return period is reduced from 100 years to 50 years, the peak ground acceleration and so also the design forces on the structure can be reduced to a fraction 0.12/0.19 = 0.60. How these ideas can be applied to retrofitting of existing bridges will be discussed in a separate section. As a matter of clarification, the "annual probability of exceedence" is merely the inverse of the "average return period".

#### Elastic response spectra

Figure 3 depicts a typical, normalised, smoothened, elastic acceleration response spectra for a given location. "Normalised" means that for the period T = 0.0 secs, which represents an infinitely rigid structure, the acceleration response (which in this case is also equal to the maximum ground acceleration in such case) has been assumed as unity and all other ordinates have been adjusted accordingly. It is interesting to note that the ordinate for 5 percent damping at the peak spectral acceleration is about 2.5 times the maximum ground acceleration<sup>34,56</sup>.

# Codification in India Background

While considerable research and codification of seismic design has been done in relation to buildings, the same quantum of effort and interest is not apparent for bridges. The structural form of bridges is different from those of buildings, including the nature of loading and the potential of dissipating energy either through ductile detailing or by external mechanical devices. Whereas the most vulnerable portion of a building is its superstructure, it is the sub-structure and foundation that are most susceptible to damage in a bridge during an earthquake. Earthquake forces on bridges are essentially codified in three separate publications in India, namely, IS:1893<sup>9</sup>, IRC:6<sup>10</sup>, and Bridge Rules<sup>11</sup>. The IS:1893 could be called the "mother code as it incorporates the zoning map of the country, design coefficients for different zones, the design criteria as well as a separate chapter for bridges. The other codes, namely IRC:6 and Bridge Rules by and large reflect the thinking of IS:1893. Other publications include IS:4326<sup>12</sup>, IS:13935<sup>13</sup> and IS:13920<sup>14</sup> of which the first two particularly and the last one largely is applicable to buildings.

The seismic coefficients specified in IS:1893 have been fixed essentially by engineering judgement. The practice in Assam before the code was originally published in 1962 was to design structures for a coefficient of  $0.08^{15}$ . These structures withstood the 1950 earthquake of Richter Magnitude 8.3 (Modified Mercalli Intensity IX). The above facts formed the basis of arriving at the design coefficients. The coefficient of 0.08 was adopted as the maximum value for the country and designated as Zone V. For other areas, the coefficients were fixed again by judgement as 0.05, 0.04, 0.02 and 0.01 for Zones IV, III , II which correspond to Modified Mercalli Intensity of VIII, VII, VI and V, respectively.

The code provides guidelines for the simplified "seismic coefficient" approach as well as for the "response spectrum" method. In the former approach the basic horizontal seismic coefficient has been fixed as indicated earlier for the various zones. For the latter method, the average acceleration spectra has been given for various values of damping as well as multiplying factors to its ordinate (termed as "seismic zone factors") which have been arrived at in such a manner that in the short period range the seismic coefficient derived from spectral considerations would be the same as the basic seismic coefficients mentioned earlier.

IS:1893, in its introduction, indicates that the maximum seismic ground acceleration cannot be precisely predicted



Figure 2. Peak ground acceleration versus return period for different seismic zones

#### Table 1. Reduction factor applicable to buildings (ATC-3)

Type of structural system	Vertical seismic resisting systemeters	em
<b>Bearing wall system :</b> Structural system with bearing walls providing support for all, or major portions of, the vertical loads Seismic force resistance is provided by shear walls or braced frames.	Light framed walls with shear panels 6 Shear walls : Reinforced concrete 4 Reinforced masonry 3 Braced frames Unreinforced and partially reinforced masonry shear walls 1	1/2 1/2 1/2
Building frame system: A structural system with an essentially complete space frame providing support for vertical loads Seismic force resistance is provided by shear walls or braced frames.	Light framed walls with shear panels Shear walls : Reinforced concrete 5 Reinforced masonry 4 Braced frames Unreinforced and partially reinforced masonry shear walls 1	7 1/2 1/2 5
Moment resisting frame system : A structural system with an essentially complete space frame providing support for vertical loads Seismic force resistance is provided by ordinary or special moment frames capable of resisting the total prescribed forces.	Special moment frames : Steel Reinforced concrete Ordinary moment frames : Steel 4 Reinforced concrete	8 7 1⁄2 2
Dual System : A structural system with an essentially complete space frame providing support for vertical loads A special moment frame shall be provided which shall be capable of resisting at least 25 percent of the prescribed seismic forces The total seismic force resistance is provided by the combination of the special moment frame and shear walls or braced frames in proportion to their relative rigidities	Shear walls Reinforced concrete Reinforced masonry 6 Wood sheathed shear panels Braced frames	8 5 ½ 8 6
Inverted pendulum structures : Structures where the framing resisting the total prescribed seismic forces acts essentially as isolated cantilevers and provides support for vertical load	Special moment frames :   Structural steel 2   Reinforced concrete 2   Ordinary moment frames : 5   Structural steel 1	1/2 1/2

with accuracy either on a deterministic or on a probabilistic basis. The code is presently under revision and a fresh look at the seismic zoning of the country has been entrusted to an expert group. The 1993 earthquake in Latur, Maharashtra, has added a new dimension to the uncertainty in seismic zoning, as this location is presently categorised under Zone I (the lowest from the point of view of ground acceleration).

It is generally believed that IS:1893 is based on a "design basis" earthquake which is about one half of the "maximum credible" earthquake; the latter being defined as the most severe ground motion that can be expected to occur at a location considering past events and geological evidence.

#### **Reduction factor**

The seismic coefficients specified in the code represent a compromise between safety and cost of the structure. It can generally be interpreted that an invisible reduction factor of about 5 has been applied to the predicted peak spectral

Table 2. Reduction factor applicable to buildings (ATC-6)

(A) Sub-	struc	ture and foundation types	
1.	RCC pile-pile cap system		
	a)	Vertical piles exclusively	3
	b)	System using batter piles	2
2.	Wall-type pier		
	a)	Weak direction	3
	b)	Strong direction	2
3.	Columns		
	a)	Single (with no redundancy)	3
	b)	Multiple (forming portals/bents)	5
(B) Connection of/with sub-structures			
1.	Superstructure to abutment 0.8		
2.	Superstructure to piers/columns 1		

accelerations of the "design basis" earthquake to obtain the seismic coefficients. The reduction factor is best justified if we keep in view the potential of energy dissipating capability of the structure as it would be prohibitive to design structures to behave elastically at peak accelerations. The reduction factor is attributable to "ductility" of the structural system, a term widely used to describe its energy dissipating capability by cyclic inelastic deformation without impairing its vertical load carrying capacity. The reduction factor has been so adjusted that structures analysed on the basis of "elastic" or "linear" behaviour for the reduced level of loading would not collapse during the design seismic event, when designed by the working stress method.

1

Substructure to foundations

3.

Unfortunately, assigning of a reduction factor is not a simple matter.

A brittle structure should have a low reduction factor (close to unity) while a very resilient and ductile system could have a high value (say 8) of the same factor. IS:1893 has identified a "Performance Factor" for buildings to introduce the element of ductility in the evaluation of seismic forces. While the "Performance Factor" values indicated for buildings itself need updating, there are no guidelines available for bridge structures.

The dire and immediate need to differentiate between buildings and bridges with respect to the reduction factor can be highlighted by comparing the recommendations of ATC- $3^3$ and ATC- $6^4$  both of which are based on the same overall design philosophy for earthquake resistant structures. Extracts are shown in Tables 1 and 2. In the case of buildings, the reduction factors vary from 1.25 to 8 for different types of structural systems. On the other hand, for bridges, the reduction factors vary from 2 to 5 for various types of sub-structure and foundations and from only 0.8 to 1.0 for connections.

At this stage, we must turn to Figure 4, which depicts typical inelastic acceleration response spectra, and compare these



Figure 3. Typical acceleration response spectra

with Figure 3, which is the elastic counterpart. The reduction factor should reflect the ductility  $\mu$  of the system and the latter is conveniently defined as the ratio of the displacement at failure to the displacement at the onset of yielding (that is, when "elastic" or "linear" behaviour is no longer applicable). After the onset of yielding, the stiffness of the structure reduces, as a result of which its natural period is lengthened and consequently the inelastic action of medium and long period structures causes a shift away from the period range of maximum response, Figure 3. We already know that for an infinitely rigid structure, T = 0, the structural response will be the same as the ground acceleration irrespective of the ductility provided in the system. In Figure 4, all the curves merge at one point at T 0, whatever the ductility of the system. Hence, for short period structures, the reduction factor should be smaller than that for structures of medium and long period range.

#### Permissible stresses and load combinations

In codal provisions, for the sake of simplicity, it is not possible to give different reduction factors for structures with different natural periods. However, it is indeed possible to stipulate that short period structures should be designed for earthquake forces at least equal to the peak ground acceleration.

The working stress method still remains the basis of design of bridges in India. For working stress design, in load combinations involving earthquake forces, the permissible increase in allowable stresses in materials is one-third as per IS:1893. The permissible increase in allowable bearing pressures for foundations is 25 percent to 50 percent depending upon the foundation type and soil strata. Certain recommendations concerning scour depth and live loads for highway and railway bridges are also given in the chapter on bridges. The provisions are deficient as they do not take into account the large number of types of loads and load combinations that have to be considered in bridge design for service loads. Reference may be made to IRC:6 to understand the complexity of the issues involved and also the contradictions in the two codes.

The codal provisions should have a transparency such that engineers are able to perceive how the "expected" ground motions have been translated into forces on the structure. It is hoped that the revised version of IS:1893 which is presently being undertaken will succeed in bringing about such a transparency which the code has lacked in the past.

# **Restraining features**

## Purpose of restraining features

Superstructures, by themselves, usually have adequate strength to resist seismic forces. However, in many earthquakes, it was noticed that the superstructure was either dislodged and fallen to the ground or was damaged due to loss of support.

To counteract such failures two specific issues need to be addressed:

- 1. Provide seismic devices for positive attachments of superstructure to pier/abutment cap.
- 2. Provide adequate support lengths for superstructure at pier/abutment cap.

Attachments of superstructure to pier/abutment cap require to be designed for some 3 to 5 times of that force for which the bridge has to be designed for from global considerations, Table 2. The reason for such a high design force is that attachments are rigid features with little or no potential for ductile behaviour during seismic action.

Friction cannot be considered as providing adequate positive attachment' due to its unreliable restraint during an earthquake.

#### **Seismic attachments**

Two examples of restraining features in projects with which the author was connected are shown in Figures 5 and 6. The first project is a 3-span continuous bridge being built by cantilever technique at Ranjit Sagar Dam project near Pathankot. It is of interest to note that the longitudinal restraint is provided only at pier P2 while transverse restraint is provided on all the supports. The second project constructed for the Algerian Railways consists of two precast girders connected with a cast-in-situ slab forming a simply supported



Figure 4. Typical inelastic response spectra



Figure 5. Seismic attachments for a continuous superstructure

bridge<sup>16</sup>. One end of each span is restrained for translation in the longitudinal direction by short vertical and horizontal cables. The other end is free to translate in the longitudinal direction. While rotational capability exists at both ends, the span is fully restrained in the transverse direction. A series of elastomeric bearings acting against projections from pier/abutment caps act as buffers to eliminate or reduce displacements.

All such seismic attachments do not have the same life span as that of the bridge and should therefore be designed and detailed in a manner that they are replaceable within the life time of the bridge.

## **Support lengths**

Adequate support length at pier/abutment caps is essential to avoid loss of support during an earthquake. Minimum support lengths as recommended in ATC-6 are reproduced in reference<sup>1</sup>. It must be cautioned that the support lengths must in addition also be checked to ensure that they would be able to accommodate displacements resulting from the overall inelastic response of the bridge, possible out-of-phase displacements of adjacent sub-structures, and possible settlements and rotation of foundations.

Some mechanical devices are in service that restrict

displacement during strong earthquakes<sup>1</sup>.

# Seismic protection for bridges Objectives of protection

Barring unusual bridges, the fundamental period of vibration of a majority of bridge structures is in the range of 0.2 to 1.2 secs. In this range, the structural response, Figure 3, is high because it is close to the predominant periods of earthquakeinduced ground motions. If the fundamental period of the bridge were to be lengthened or if the energy dissipating capability of the structure were to be increased, or both, the seismic foices on the structure could be reduced. Seismic protection devices have essentially been developed in Japan, the USA, New Zealand and Italy, keeping in view these two options available for reducing the lateral inertial force of the structure due to seismic activity.

In most of the applications of seismic protection, a mechanical device is introduced below the superstructure so as to "isolate' it at the top of the pier/abutment cap. The objective is two-fold: reducing seismic loads, and, distributing the total seismic load of the superstructure equally to the various substructures of the bridge.

## **Fluid damper**

One of the devices used for reducing bridge response is the fluid damper which provides a resistive force when subject to "instantaneous loads" such as earthquake and wind but which does not provide constraint to "slowly applied loads" like change of temperature and creep and shrinkage of concrete. The principle of the fluid damper is explained in Figure 7. The ends A and B may be connected to the bridge as follows :

- 1. End A to superstructure and end B to substructure for continuous spans or for simply supported spans
- 2. Ends A and B to adjacent superstructures over the pier for simply supported spans.

While fluid dampers have been used in Japan for a fairly long time, it is recognised that their maintenance requires special attention periodically. Also, they occupy not an inconsiderable space on the pier/abutment caps.

## **Elastomeric bearings**

Various types of elastomeric bearings can be gainfully employed as seismic isolation devices by elongating the period of vibration of the structure.

The normal elastomeric bearings as per IRC:83 (Part II)<sup>17</sup>, may not be appropriate as this code does not envisage the use of elastomeric bearings as seismic isolation devices. It is necessary to investigate experimentally the equivalent stiffness and damping factor of the loaded bearing to fully reversed cycles of maximum expected displacement at the fundamental period of the structure. The acceptability of the bearing as well as the data for its design can be determined only after such a test has been performed.



Figure 6. Seismic attachments for a simply supported girder bridge

While elastomeric bearings of traditional construction with embedded steel plates have their utility in some cases, the displacements of the superstructure in relation to the pier/abutment caps during the design earthquake often become excessive. The displacement response can be reduced significantly by the use of "Lead Rubber Bearings<sup>11</sup>. (LRB) and "High Damping Rubber Bearings" (HDR), depicted in Figure 7. The LRB has the same type of construction as the traditional elastomeric bearings but incorporate one or more lead plugs to increase the damping and hence reduce the displacement of the superstructure. The same objective is achieved by the HDR bearing which utilises special energy-absorbing rubber.

#### Seismic fuse

Some other types of seismic protection devices have been discussed in reference<sup>1</sup>. Of special interest is the incorporation of a "seismic fuse" in a bridge. This technique involves acceptance of significant localised damage or intentionally designing an element of the bridge to fail during an earthquake of a pre-determined intensity. After such a local damage or failure, that is, "blowing" of a seismic fuse, the dynamic behaviour of the remaining structure is pre-designed to alter in such a fashion that it suffers the least impairment. Sacrificial elements could include a part of dirt wall of the abutment, a designated portion of the deck slab, a "stopper" restricting the deformation of bearings, etc.

Increased displacement of superstructure with respect to the pier/abutment cap is inherent in all cases of seismic protection discussed above, that is, either by mechanical devices or by seismic fuse. Hence, expansion joints for the expected movement require careful consideration.

Properly designed seismic protection devices have not yet been used in India. A collaborative effort between designers, manufacturers and testing agencies is required to develop such devices which are suited to Indian conditions.

# Bridge retrofitting for earthquakes Need of retrofitting

"Retrofitting" is a term used for strengthening of the elements of an existing bridge or modifying the dynamic behaviour of the existing bridge so that it is able to withstand future earthquake shocks of a specified intensity.

Retrofitting of a bridge may be considered necessary for upgrading the capacity of existing bridges for a variety of reasons. Predictions of ground accelerations due to seismicity may change over a period. It has often been noticed that most old bridges which are still serviceable for vertical loads were not specifically designed for earthquake forces.

#### **Retrofitting strategies**

Retrofitting techniques generally revolve around one or more of the following strategies :

- 1. Improvement of restraining features of the superstructure so that it is not dislodged from its bearings.
- 2. Strengthening of the sub-structure and foundations, and/or improving their ductility.
- 3. Adjusting the seismic response of the structure by replacing existing bearings and/or installing special mechanical protection devices.

In many cases the decision due to economic factors may be to "carry the risk" and meet the expenditure after the damage has occurred. While a proper cost-benefit analysis is still a far-cry for seismic retrofitting of bridges because of various imponderables, a priority rating of the flock of bridges requiring such attention is almost always attempted after undertaking condition surveys.

## Design criteria for retrofitting

Whenever retrofitting of a bridge is contemplated, the most important question to be answered is : what should be the design earthquake forces ? It is not necessary to upgrade an existing bridge to have the same level of seismic resistance as is specified for the new bridges. The answer to the question can be best answered by the example that follows.

Assume that present norms of earthquake resistant design are based on a design life t of 100 years with a probability of exceedence p of 0.5. From equation (1) or Figure 1, the return period of the earthquake can be evaluated as 150 years. A bridge constructed 50 years ago was originally designed without taking into account any seismic forces. The bridge



Figure 7. Seismic protection devices

now needs to be retrofitted and it has been determined that it falls in a zone of moderate seismicity (say, Zone III of IS:1893). We require to evaluate what forces it should be designed for in comparison to a new bridge located in the same seismic zone.

The remaining life of the bridge is 100 - 50 = 50 years. With p = 0.5, the return period ts can be evaluated as 75 years with the help on Figure 1, or equation (1). Now reverting to Figure 2, the return periods of 75 years and 150 years on the curve of moderate seismicity have ordinates in the ratio of 0.17:0.27. Hence, the design earthquake forces for retrofitting of the existing bridge would be about 63 percent of those for which a new bridge in the same seismic zone would be designed for.

# Conclusions

The structural form of bridges is different from those of buildings, including the nature of loading and the potential of dissipating energy through ductile detailing or by external mechanical devices. The present Indian codes for seismic design do not consider this issue with the attention it deserves.

Seismic devices for positive attachment of superstructure to pier/abutment cap and adequate support length for superstructure at pier/abutment cap are of paramount importance. The connections are to be designed for 3 to 5 times of that force for which the bridge is designed from global considerations. Seismic protection devices for bridges can be used effectively to alter the dynamic behaviour of the bridge and thereby increase its energy dissipating capability and/or lengthen its natural period. Properly designed devices require to be developed which would suit Indian conditions.Seismic retrofitting techniques are used for strengthening or modifying the dynamic behaviour of the bridge. The techniques can be used for bridges in which damage has already occurred or for upgrading the capability of bridges to withstand future shocks. It is not necessary to design the strengthening/upgrading measures for the full design criteria.

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