Design of pile caps – Strut and tie model method

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The origin of Strut and Tie Models for detailing reinforced and prestressed concrete structures can be traced back to the study of the behaviour of reinforced concrete elements subjected the shear, as early as 1899 by Ritter[1]. Since then it has gained popularity in Europe for evolving practical reinforcing details in a variety of situations. Appendix A of American Concrete Institute Building Code, ACI 318-2005 is entirely devoted to Strut and Tie Model (STM) method[2]. There are several situations where there is no alternative to STM for detailing reinforced or prestressed concrete structures. This is not always the case with pile caps. Even so STM gives a better insight to structural behaviour of pile caps especially when they are deep. An attempt is made to evaluate the STM approach to design of pile caps supporting 2, 3 or 4 piles.

Strut and tie model method (STM)

Every concrete structure whether reinforced or prestressed can be divided into B regions where beam like behaviour is valid and D regions where beam type behaviour gets disturbed and there are local concentration of stresses, Figure 1. It is useful to remember that at supports, at beam column junctions, at points of application of concentrated loads, brackets represent D regions.

In corbels and deep beams there are no B regions left and hence linear strain variation is disturbed in the entire element.

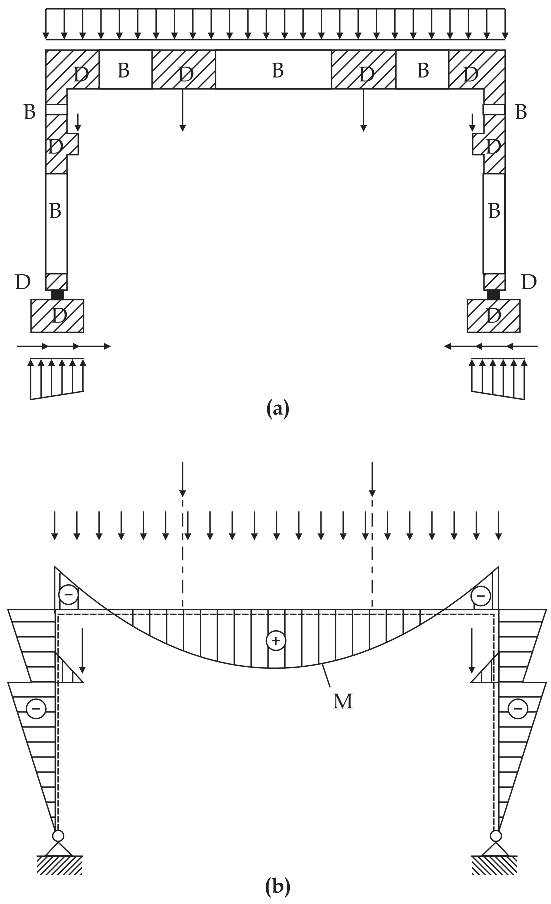

Figure 1. (a) Framed structure showing disturbed regions (b) Analytical model

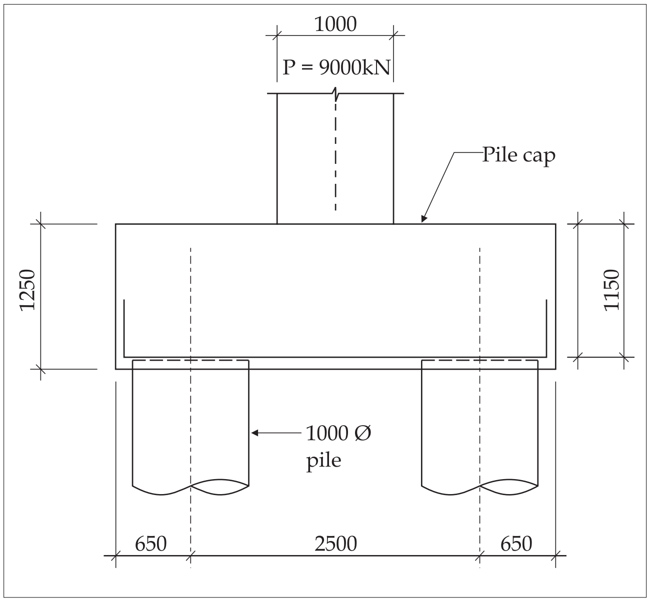

Figure 2. Pile cap supported by two piles

Pile spacing, depth of pile cap and STM for pile caps

Piles are spaced as close as permitted by geo technical considerations (2.5 times the diameter for end bearing piles or 3.0 times the diameter for piles transmitting the load by skin friction). When bending moments are large it may become more economical to increase the spacing of the piles. Indian Road Congress, IRC-21, specifies that if STM approach is used for design of pile caps the thickness of the pile cap should not be less than 0.5 times the pile spacing[3]. And if the piles are spaced more than three pile diameters, IRC 21 recommends that only the reinforcement placed within 1.5 pile diameters shall be considered to constitute a tension member. Also 80% of the tension member reinforcement shall be concentrated in strips linking the pile heads. No check for shear is required to be carried out for pile caps designed and detailed according to STM methods.

Two pile group

Consider a simple pile cap supported by two piles subjected to a vertical load.

$M_{face} = 4500 \times (1.25 - 0.50) = 3375$ kNm (T = 3264 kN)

$M_{max} = 4500 \times (1.25 - 0.25) = 4500$ kNm (T = 4348 kN)

If the structure shown in Figure 2 represented a floating column then one would have used M_{max} without thinking twice. But, then what is the rationale in using M_{face} for pile cap? Incidentally, both IRC 21 and IS 456 state that critical section for bending moment in pile cap shall be taken as face of columns.

Considering that the span to depth ratio is smaller than 2, let us try deep beam approach. The clear span replacing 1 m diameter circular section by an equivalent square section of 0.89 m, the effective span as per IS 456 will be:

$1.15(2.5 - 0.89) = 1.85$ m

$l/D = 1.48$

$z = 0.2(1.48+2)D = 0.70\ D$

$M_{max} = 4500 \times (0.925 - 0.25) = 3038$ kNm (T = 3492 kN)

It may be concluded that designing for face moments and beam like behaviour does not appear to be a conservative approach.

STM method for pile cap with two piles

The finite size of the column can be considered by splitting the vertical load into two equal parts. The effective depth of the truss is assumed to be 1.035 m. Tensile force can be estimated by resolving forces. Width of pile cap is 1.3 m.

T = 4500 / tan 45.98° = 4348 kN

Reinforcing steel will be provided corresponding to this tensile force.

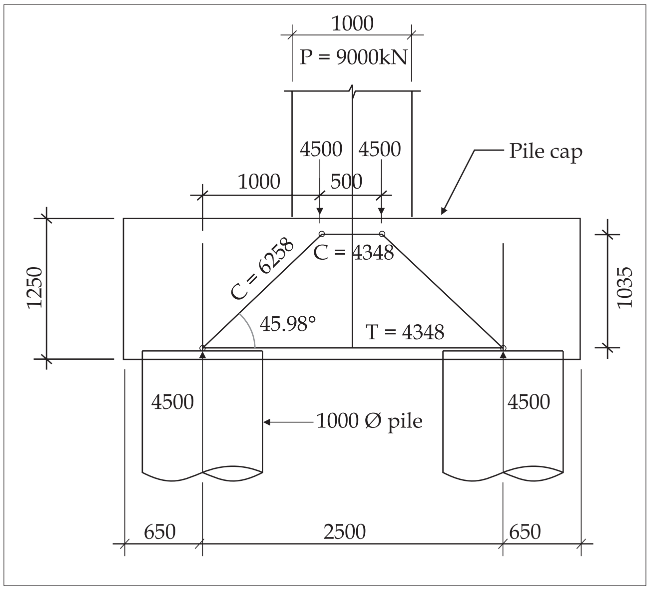

Figure 3. STM model for pile cap with two piles

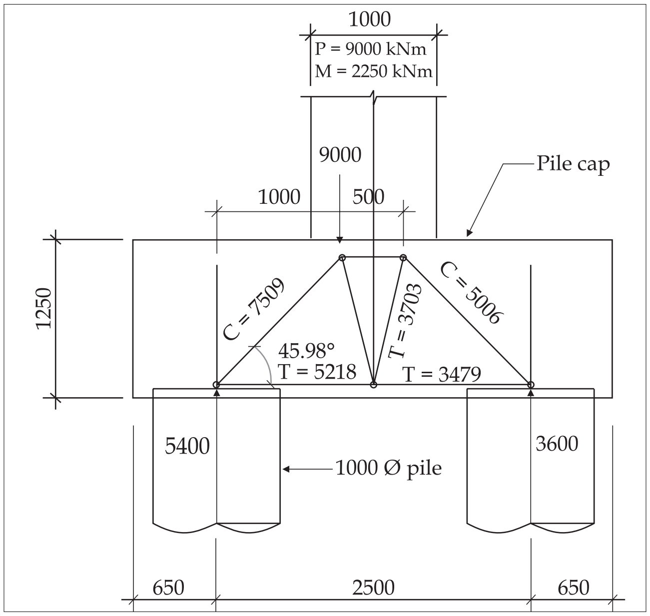

Figure 4. Axial load with moment (not appropriate)

If we had used the bending theory the corresponding tensile force assuming a beam like behaviour would have been

T = 4348 kN (adopting maximum moment)

T = 3264 kN (adopting face moment)

When we use STM we will have to provide full development length beyond the centre line of the pile. Thus, it is seen that by using face moments as permitted in the codes of practice and ignoring deep beam behaviour, we are in fact providing much lesser reinforcement than what would be required by adopting STM approach.

Consider a factored bending moment of 2250 m-kNm applied at the top of pile cap then the maximum reaction on the pile will be 5400 kN. If we continue to use the same STM model additional members have to be introduced so that no mechanism is formed. It is found that a new tie with a force of 3703 kN is required to be introduced which seems to be surprising at first glance, Figure 4.

Now, consider another possible model shown in Figure 5. With this STM it is seen indeed that additional tensile tie force is no longer necessary. The point is that one STM is not valid for all load cases. As will be explained later, the STM shown in Figure 5 is the appropriate choice.

It is clear from the foregoing discussion that a satisfactory STM model depends not only on the structural configuration but also on the type of loads. Today we are in a situation where even the simplest of structures are checked for no less than perhaps 20 load combinations which virtually could mean several STM models for the same pile cap. But, by using judgment it will be observed that the maximum tie force is more or less linearly related to the maximum pile reaction.

If the bending moments are so large that tensile stresses are generated in the column reinforcement then the equivalent loads used, have to be accordingly modelled. But if the pile goes in tension then an altogether different model will have to be looked into.

Columns are also subjected to lateral loads resulting in bending moments in piles. The strut and tie model gets even more complicated in such situations. Often bending moments in piles are quite small and nominal reinforcement (shrinkage/temperature reinforcement) is adequate to take care of these bending moments. So far the discussion has been confined to tie forces that will determine the amount of reinforcement.

Guide lines for STM methods

The design of disturbed region (D-region) can be based on finite element analysis but the major problem is to arrive at practical reinforcement layout. Lever arm of uncracked sections is always less than that of cracked sections. We could always take advantage of this fact while using uncracked FEM analysis as a basis for STM methods. Another option is to follow the load path.

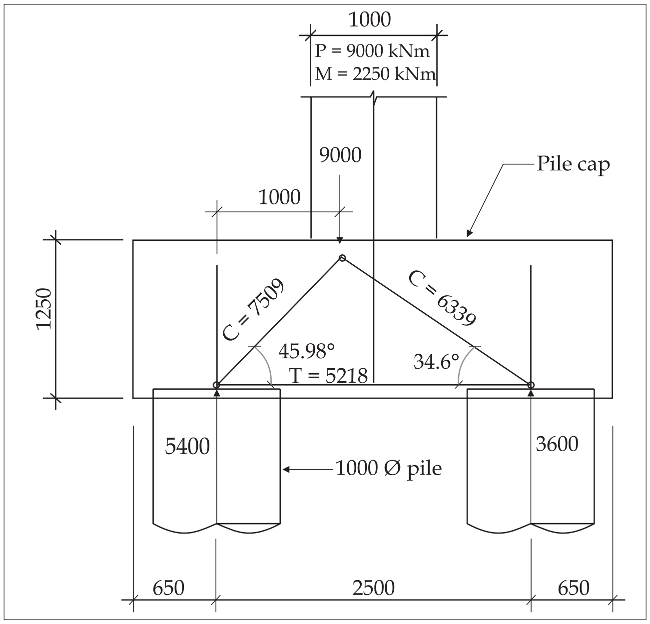

Figure 5. Axial load with moment (appropriate)

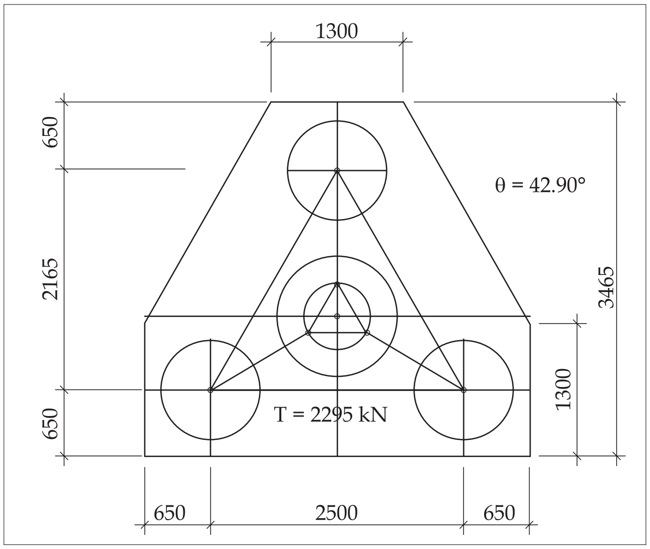

Figure 6. STM model for 3 pile group

However, detailing based on models that deviate too much from the elastic behaviour are susceptible to wide cracks. Developing a suitable STM model in such cases is very instructive. It requires training and experience with the STM methods. A systematic approach is needed and hastily drawn models may satisfy neither equilibrium nor compatibility conditions.

Basically, STM model will include struts, tie and nodes. As we have already seen there are multiple STM models that satisfy equilibrium condition. It is useful to remember that the structure tries to carry loads as effectively as possible with the least amount of deformation. Since the contribution of the tensile forces to displacement is much more than that of concrete struts, a model with shortest ties and least tie forces is the most effective. Applying this principle, and comparing the models in Figures 4 and 5, we can conclude that Figure 5 indeed is the appropriate model.

Strut and tie model demand much more involvement from the designer compared to computer analysis. It is instructive and helps in avoiding major mistakes. Without doubt the modelling process is not a unique solution, which is considered by some as a major draw back of STM approach.

We should look for simple models with a small numbers of struts and ties, following the directions of principal stresses. Since STM is dimensioned for factored loads, understanding elastic behaviour is essential for providing guidance for evolving sound details that satisfy serviceability criteria.

Angles between struts and ties should be at least 45° whenever possible. Exception from this rule is when a diagonal compression strut meets two ties in orthogonal direction. Angles smaller than 30° are unrealistic and involve high compatibility strains (ACI 318 permits angles up to 25°).

Strength of concrete compression fields

The stress assumed in the compression field has been elaborated in FIB Bulletin 3[4] which incidentally is also adopted by ACI 318 : 2005[2]. This is given by the following equation:

$$f_{cd,\,eff} = v\,(1 - f_{ck.cyl}/250)\, f_{1cd}$$

where

$f_{ck.cyl}$ = characteristic cylinder strength

f_{1cd} = design strength = $0.85\, f_{ck.cyl} / 1.5$

Since our codes refer to cube strengths

$$f_{cd,eff} = v\,(1 - f_{ck,cube}/300)\; 0.45\, f_{ck,cube}$$

where

v = 1 for uncracked sections
= 0.80 for struts with cracks parallel to strut and bonded transverse reinforcement
= 0.60 for struts transferring compression across cracks with normal crack width
= 0.45 for struts transferring compression across large cracks (members with axial tension, or flanges in tension)

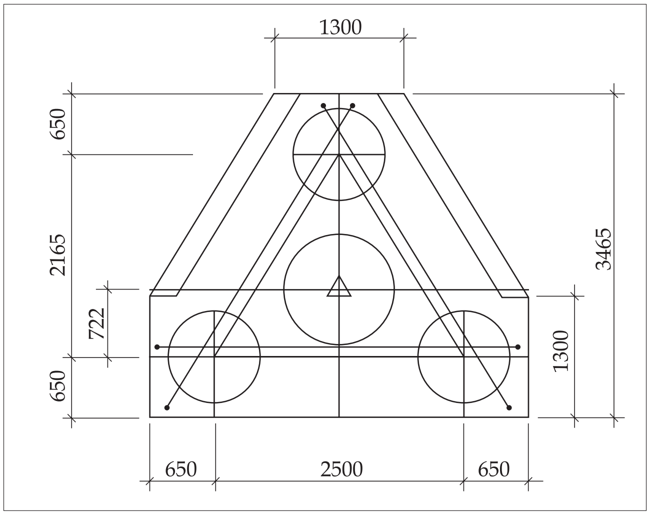

Figure 7. Reinforcement layout for 3 pile group

Reinforcement parallel to compression struts should be considered provided that they are sufficiently secured against buckling.

Strength of steel tie (reinforcing steel)

This is based on the yield stress of steel divided by partial safety factor of 1.15. For grade Fe 415 the effective stress will be 361 MPa.

Nodes and anchorages

Nodes are points where struts and ties meet. These are classified by the types of forces that meet at the node.

- CCC Three struts meet at the node
- CCT Two struts and one tie meet at the node
- CTT One strut and two ties meet at the node

The nodes shall be dimensioned and detailed so that all the forces are balanced and any other remaining ties anchored or spliced securely. The nodes must be generally verified for:

- Anchorage of ties in the node
- Compressive stress in the node

For biaxial compression the permissible stress can be increased by a factor of 1.20. For CCT or CTT nodes compression check is often not critical for pile caps but if a check is required, a reduction factor of 0.80 should be applied considering cracking due to tension induced by anchorage of bars.

Let us check depth of compressive stress field in the horizontal strut assuming concrete to be of M40 grade, Figure 3.

$F_{cd,eff} = 0.867 \times 0.45 \times 40 = 15.6$ MPa

$x = 4348/1000/15.6/1.30 = 0.214$

Depth of truss $= 1.15 - 0.214/2 = 1.043 \approx 1.035$

Stress in diagonal strut for a 1.0 m diameter pile section.

$f = 6258/1000/0.785/\sin 45.98^{\circ} = 11.1 \text{ MPa} < 15.6 \text{ MPa}$

The above calculation is approximate. A more exact calculation will involve bottle shaped struts.

As far as the node is concerned

$f = 4500/1000/0.785 = 5.73$ MPa

Thus, it is seen that compressive stresses are very much within the permissible values.

Since STM only deals with limit state of collapse, it will be necessary to provide supplementary face reinforcement and shrinkage reinforcement.

STM for a 3 pile group

Consider a three pile group supporting a factored column load of 13500 kN, Figure 6. Assuming the pile cap to be 1250 mm deep, column diameter as 1200 mm and assuming the effective depth to be 1.035 m, then the angle of the strut will be 41.1° if the size of the column is neglected. However, considering the finite size of the column the angle of the inclination of the compressive strut (θ) to the horizontal plane will be equal to 42.9°.

$T = 4500/\tan 42.9^{\circ}/2/\cos 30^{\circ} = 2796$ kN

$C = 4500/\sin 42.9^{\circ} = 6611$ kN

The reinforcement layout using this approach is also shown in Figure 7.

Effect of bending moments on the column can be dealt by splitting the column loads into equivalent loads and relating the tie force to the maximum pile reaction.

STM for a 4 pile group

Consider a four pile group supporting a column load of 18000 kN. Let us assume a column size of 1200 mm and divide the load into four equal parts for a 1250 mm deep pile cap. The effective depth is taken as 1.035 m and θ = 37.6°. Then,

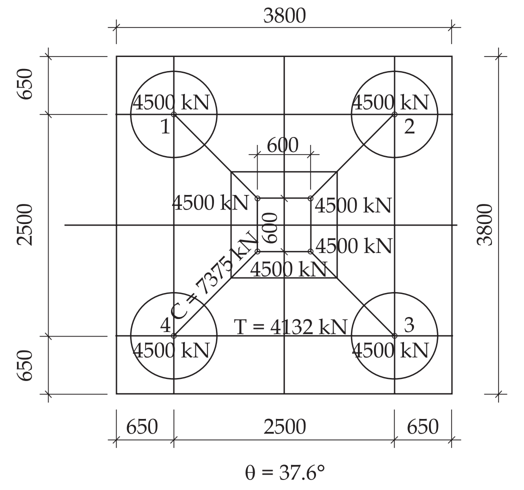

Figure 8. STM for 4 pile group

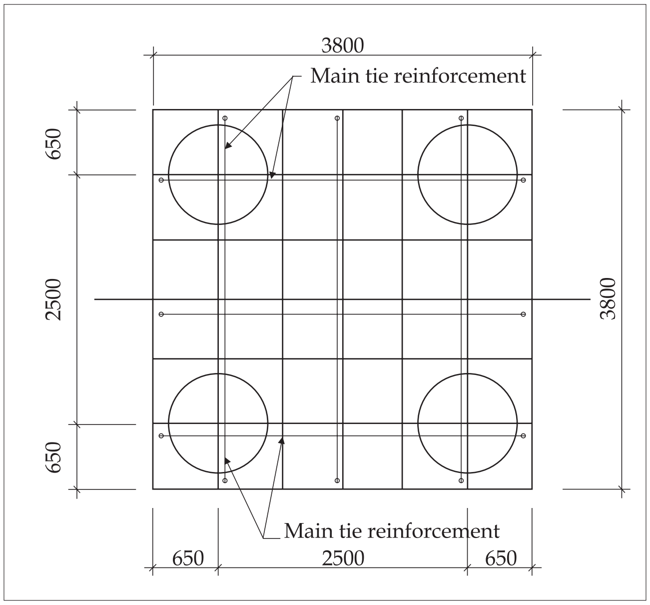

Figure 9. Reinforcement layout

T = 4500 /Tan 37.6° × Cos 45° = **4132** kN

C = 4500/Sin 37.6° = 7375 kN

If face moments had been used and beam like behaviour assumed, then reinforcement would have been provided for a tensile force of 4130 kN. Once again it is seen that there is a considerable increase in reinforcement by using STM model. The reinforcement layout using STM approach is shown in Figure 10.

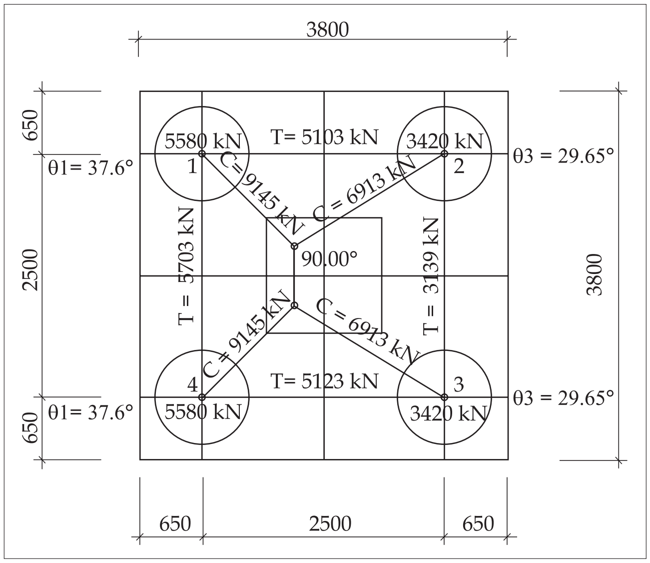

Figure 10. 4 pile group with axial load and moment

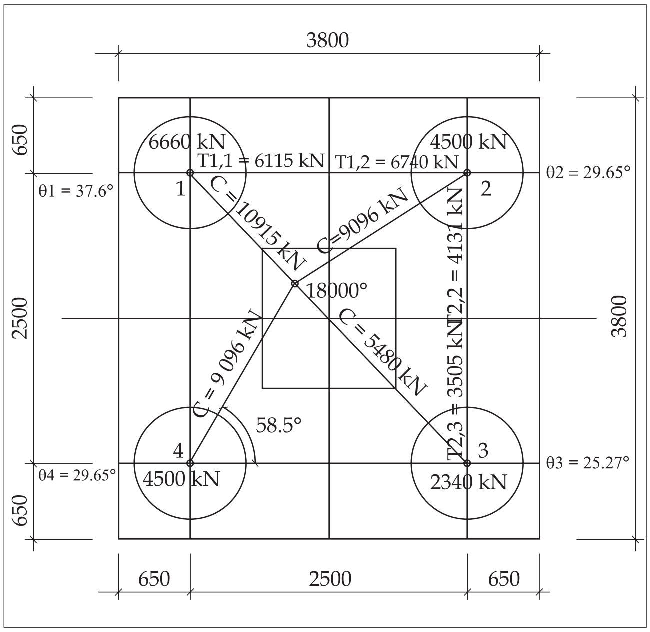

Figure 11. 4 pile group with axial load and biaxial moment

Bending moments applied at the top of column can be replaced by a set of axial loads. This has to be done because struts and ties cannot resist bending moments. The distance between the equivalent loads on the column does not materially affect the results since the pile reactions are known.

Now consider a factored bending moment of 5400 kNm in one direction in conjunction with a factored vertical load of 18000 kN. Maximum and minimum pile reactions

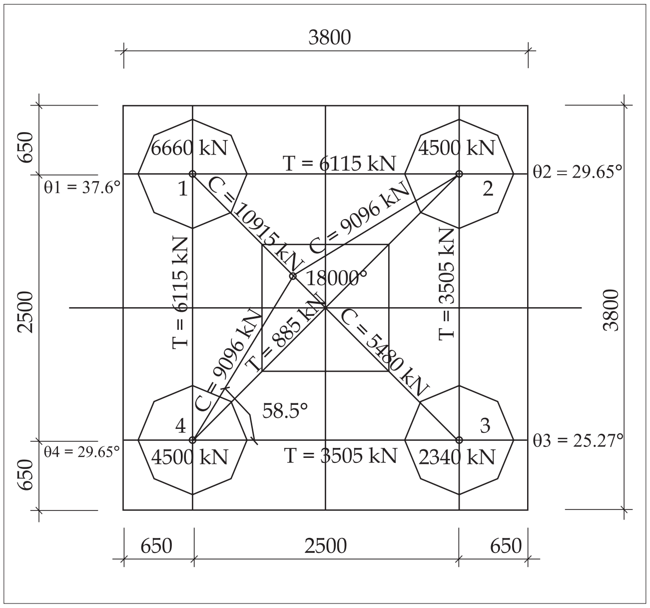

Figure 12. STM with diagonal tie

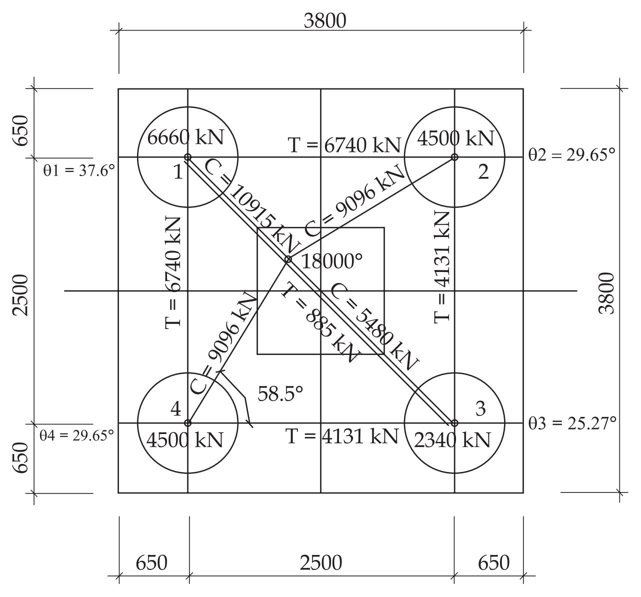

Figure 13. STM with diagonal strut

will be 5580 kN and 3420 kN, respectively, Figure 10. Splitting the loads as before, maximum tie and strut forces can be derived from the pile reactions.

Using the maximum pile reaction, the forces will be

$T_{1,1}$ = 5580/Tan 37.6° × Cos 45° = 5123 kN

C_1 = 5580/Sin 37.6° = 9145 kN

From the minimum pile reaction, the strut and tie forces will be:

$T_{1,2}$ = 3420/Tan 29.65° × Cos 31.5° = 5123 kN

C_2 = 3420/Sin 29.65° = 6913 kN

Since $T_{1,1} = T_{1,2}$ there will be no strut or tie forces in the diagonal members in the plane of reinforcement.

Consider a factored bending moment of 5400 kNm in each direction in conjunction with a factored vertical load of 18000 kN. The maximum pile reaction will be 6660 kN and the minimum pile reaction will be 2340 kN

Splitting the loads as before, maximum tie and strut forces will be

Corresponding to a pile reaction of 6660 kN

$T_{1,1}$ = 6660/Tan37.6° × Cos 45° = 6115 kN

C_1 = 6660/Sin 37.6° = 10915 kN

Corresponding to a pile reaction of 4500 kN

$T_{1,2}$ = 4500/Tan29.65° × Cos 31.5° = 6740 kN

C_2 = 4500/Sin 29.65° = 9096 kN

$T_{2,2}$ = 4500/Tan29.65° × Cos 58.5° = 4131 kN

Corresponding to a pile reaction of 2340 kN

$T_{2,3}$ = 2340/Tan25.27° × Cos 45° = 3505 kN

C_3 = 2340/Sin 25.27° = 5481 kN

Conditions $T_{1,1} = T_{1,2}$ and $T_{2,2} = T_{2,3}$ can be met with by introducing a diagonal member at node 1 (Figure 13) or at node 2 (Figure 12). Strut at node 1 (Figure 13) is the preferred option because of less number of ties.

Conclusion

From the forgoing discussion it is seen that STM method for pile caps will result in more flexural reinforcement than what one would have obtained by using beam theory and face moments permitted by codes of practice. However, no shear reinforcement will be required. STM method requires reinforcement to be distributed in bands. Nominal reinforcement is required to be provided in other areas for serviceability considerations.

Same STM cannot be used for all loading cases involving bending moments. STM is a very effective and useful tool for enabling consistent detailing. It is also a very educative tool since the designer can no longer rely only on computers and will be encouraged to understand the fundamentals of structural behaviour.

References

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