Seismic response prediction of reinforced concrete single barrel shell structures by nonlinear static analysis

R. Pathak and R.K. Khare

This study deals with seismic response prediction of reinforced concrete single barrel shell structures under nonlinear static loads. Pushover analysis has been widely used on earthquake response prediction of building structures under severe earthquakes. It needs to be studied whether it is applicable for reinforced concrete shell structures or not. In this paper, pushover analysis of long single barrel shell structure is attempted. A reputed finite element software is used to perform pushover analysis. To understand the behavior of shell structure different parameters are summarized for pushover analysis in different directions.

1. INTRODUCTION

Nonlinear Static Procedure has been widely accepted as a useful tool for performance-based seismic design and evaluation of structures. In the short time that has elapsed since its introduction to the engineering community, the method has been a subject of extensive research and several new analysis approaches have been proposed. It is now common to estimate seismic demands in a simplified manner by nonlinear static analysis or pushover analysis, which seems to be the preferred method in structural engineering practice [1] [2] [3].

Pushover analysis has been widely adopted in the seismic analysis of low- and medium-rise structures; however, few research references about pushover analysis of the long-span spatial structures have been reported till now [4] [5] [6]. Meek and Wang (1998) presented a simple, effective finite element incremental formulation and procedure for geometrically nonlinear static and dynamic analysis of a shell structure with finite rotations, based on both the total Lagrangian and updated Lagrangian description of motion [7]. Zhang and Qian (2009) introduced pushover analysis of a large-span spatial structure, Beijing A380 hangar structure at Capital International Airport [8]. Boutagouga and Djeghaba (2013) focused on the effect of the in-plane rotational degrees of freedom in linear and geometrically non linear static and dynamic analysis of thin shell structures by flat shell finite elements [9]. Past research work still lacks in the nonlinear analysis of shell structures with support system to get a proper insight of the behaviour.

In this paper, pushover analysis of long single barrel shell structure is attempted. The lateral loading pattern adopted is unit acceleration in X, Y and Z direction to perform the pushover analysis. A three-dimensional finite element model for seismic analysis is developed. A complete pushover analysis is performed using the finite element software [10].

2. DESCRIPTION OF THE STRUCTURE CONSIDERED

A long single barrel shell is used in this study to cover an area of 36 m x 20 m. IS 2210:1988, Criteria for Design of Reinforced Concrete Shell Structures and Folded plates [11], suggests to design the shell structures for seismic loads in accordance with IS 1893:2002, Criteria for earthquake resistant structure [12], however lack in guidelines and specifications for nonlinear seismic analysis and design of shell structures to be adopted. The structural system is designed as per IS 1893:2002, Criteria for earthquake resistant structure, for Single Barrel Shell Structure. The design spectrum used is of medium soil as per IS 1893 Part I (2002) and the structure is considered in Zone-V. The cylindrical shell is supported on rigid diaphragm at edges in Y-direction and edge beams in X-direction. A live load of 0.75 kN/m^2 is applied on the structure. Figure 1 (a) shows the meshing view for single barrel shell structure. The radius of shell is 20 m. The thickness of shell is 0.15 m. The semi-central angle is kept

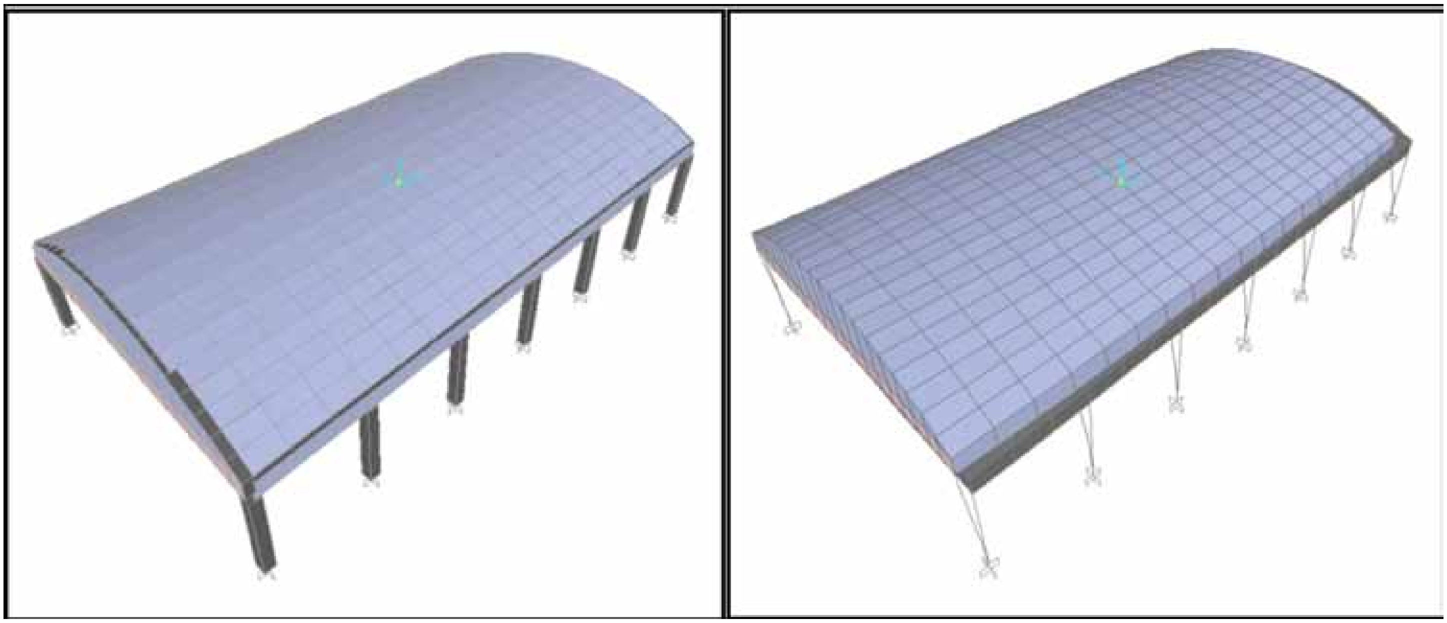

Figure 1 (a) Meshing view (b) First mode shape of single barrel shell structure

Table 1. Details of parameters for single barrel shell structure

S.No.	Description	Parameter
1.	Span	36 m
2.	Chord Width	20 m
3.	Live load	0.75 kN/m2
4.	Grade of Concrete	M-25
5.	Type of Steel	HYSD bars
6.	Column Height	6 m
7.	Column Size	0.6 m x 0.6 m
8.	Edge Beam Size	0.5 m x 1 m
9.	Shell reinforcement	10d @ 200 centre to centre at both-faces.
10.	Diaphragm thickness	0.50 m
11.	Radius of Shell	20 m
12.	Thickness of Shell	0.15 m

30° to keep radius and chord width same. All columns are 0.6 m x 0.6 m in size and 6 m in height. Shell surface and diaphragms are discritized as area elements. The edge beams are modelled as area elements for a true representation of connectivity with shell element. Columns are modelled as frame elements. Table 1 gives the details of parameters considered for single barrel shell structure.

3. FREE VIBRATION BEHAVIOR

The free vibration analysis of single barrel shell structure is performed to get a first important insight into structural dynamic properties [13] [14] [15]. The modal characteristics of the single barrel shell structure are presented in the X, Y and Z directions in Table 2 for first 10 modes. The prominent modes in the X, Y and Z directions are Mode 1, 2 and 5 respectively. Figure 1 (b) shows deformed shape of first

Table 2. Modal characteristics of single barrel shell structure

Mode	Period (s)	Frequency (Hz)	Modal Participating Mass Ratio			Mass Participation Factor		
			X	Y	Z	X	Y	Z
1	1.0859	0.9209	0	**0.9553**	2.12E-11	3.6E-11	**-39.25**	0.0002
2	1.5223	0.6568	**0.9996**	2.91E-20	6.49E-17	**40.15**	6.85E-09	-3.2E-07
3	1.6453	0.6077	8.96E-10	0	0	0.0012	3.15E-11	-1.5E-09
4	2.6454	0.3780	0	0.0387	6.35E-12	1.31E-09	7.90	-0.0001
5	2.8022	0.3568	0	1.52E-13	**0.3757**	-1.3E-09	0	**24.61**
6	4.2819	0.2335	4.09E-19	3.58E-14	0.1294	-2.6E-08	7.6E-06	14.44
7	4.6525	0.2149	8.02E-11	7.8E-18	4.49E-16	-0.0004	1.12E-07	8.51E-07
8	5.0292	0.1988	1.39E-06	3.54E-18	3.12E-15	-0.0474	7.55E-08	2.24E-06
9	5.4538	0.1833	4.17E-19	1.85E-12	0.0081	2.59E-08	0	3.62
10	5.6912	0.1757	0.0001	3.51E-17	1.1E-15	0.4015	2.38E-07	1.33E-06

mode, Figure 2 shows deformed shape of second mode and fifth mode for single barrel shell structure.

4. FINITE ELEMENT MODEL

The Finite Element model includes the shell structure and its supporting structure. The number of shell area elements is 720 and the shell is discritized in to 1m x 2m element. The shell element is idealized as an assemblage of thin constant thickness element with each element subdivided into three numbers of layers. The layered shell allows any number of layers to be defined in the thickness direction, each with an independent location, thickness, behavior, and material [16] [17]. Mander unconfined concrete model is used for concrete of M-25 Grade The unconfined properties are parameters used in defining the Mander unconfined concrete stress-strain curve of the column core [18]. Uniaxial Takeda model is used for reinforcement steel of Fe-415 grade. Material behavior is considered to be non-linear. Three-dimensional finite element modeling of the single barrel shell structure is performed using the software program.

5. NON-LINEAR STATIC ANALYSIS

A pushover analysis is performed by subjecting a structure to a monotonically increasing pattern of lateral loads, representing the inertial forces, which would be experienced by the structure when subjected to ground shaking. Under incrementally increasing loads various structural elements may yield sequentially. Consequently, at each event, the structure experiences a loss in stiffness. Using a pushover analysis, a characteristic non-linear force displacement relationship is determined.

Pushover analysis is performed for unit acceleration in three directions i.e. X, Y and Z directions. Unit acceleration is applied at the top node of crown at centre of plan. The software's static pushover analysis capabilities, which are fully integrated into the program, allow quick and easy implementation of the pushover procedures prescribed in ATC-40 [19] and FEMA-356 [20] for both 2 dimensional and 3 dimensional structures. It also provides default-hinge properties and recommends PMM hinges for columns and M3 hinges for beams as described in FEMA-356. The beams are modelled as area element for a true representation of connectivity with shell element. The formation of hinges is not possible in area elements. Therefore to incorporate nonlinearity in area elements the layered shell is provided with nonlinear material behavior. The software provides facility to define a linear as well as nonlinear layered shell area element. Cylindrical barrel shells are supported on edge beams and columns. The edge beams are provided as an area element instead of line element. The model used for pushover analysis is considering the representation of actual cylindrical shell structures with support system including edge beams, diaphragms and columns. Edge beams and diaphragms are modelled as area elements for a true representation of connectivity with shell element. The area elements are defined as layered shell with nonlinear material behavior. PMM hinges are provided in columns [21].

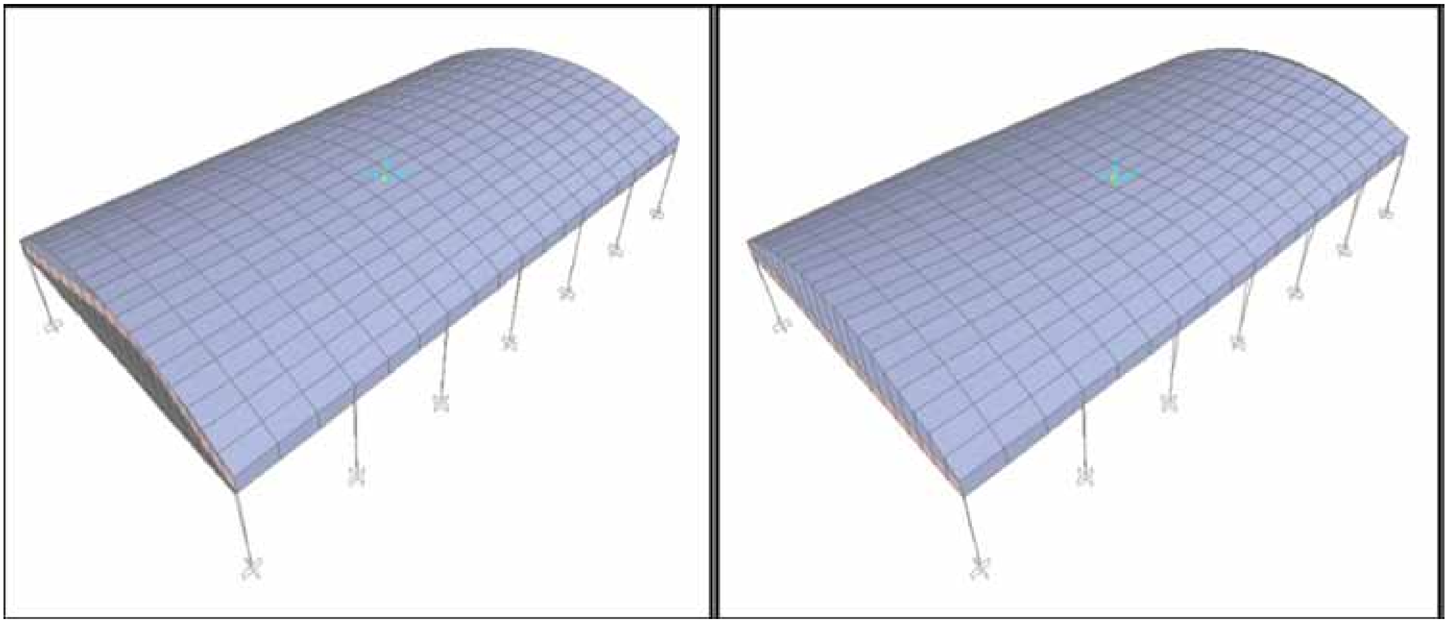

Figure 2 (a) Second mode shape (b) Fifth mode shape of single barrel shell structure

6. RESULTS AND DISCUSSIONS

The results considered from pushover analysis are capacity curves, capacity-demand curves and performance points. To understand the behavior of shell structure parameters like displacements, membrane forces, membrane stresses and shell layer stresses are determined for shell elements. Maximum positive and negative values are reported for these parameters.

1) Capacity curve

The capacity curves for the shell structure in X, Y and Z direction are shown in Figure 3. All the curves show similar features. They are initially linear but start to deviate from linearity as the columns undergo inelastic actions. When the structures are pushed well into the inelastic range, the curves become linear again but with a smaller slope. The bilinear capacity curve helps to locate the values of base shear and displacement at the yielding. The same are tabulated in Table 3. The curves show no decrease in the load carrying capacity of the buildings suggesting good structural behaviour.

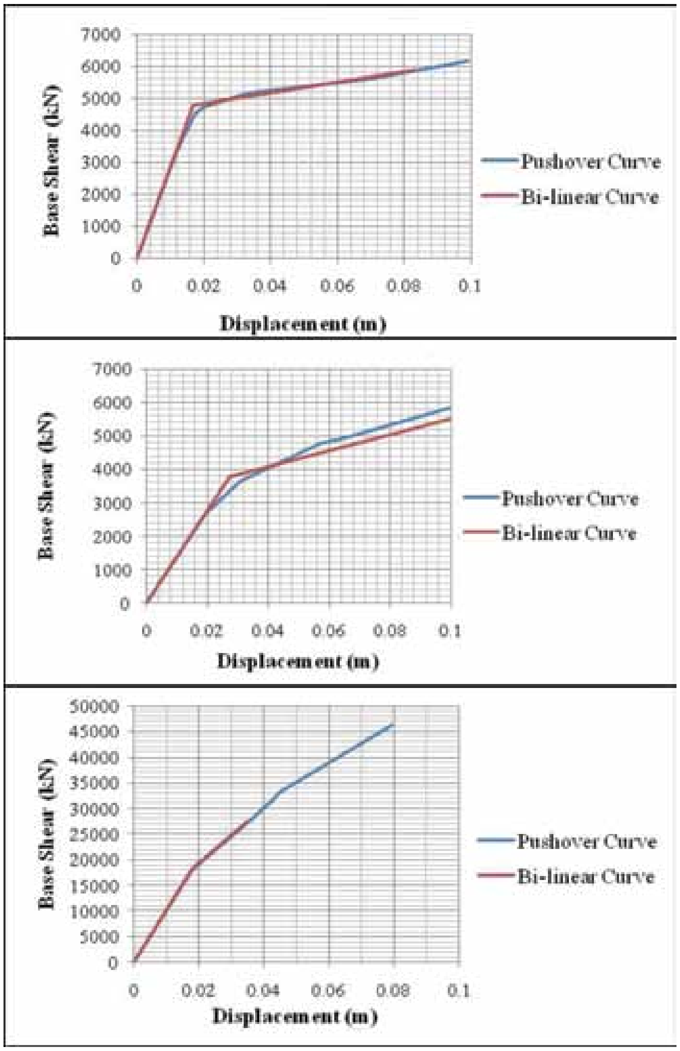

Figure 3. Capacity curve for single barrel shell structure in X, Y and Z directions

Table 3. Response of single barrel shell structure by pushover analysis

Pushover Analysis	Yield point		Performance point		Performance point	
	V_y (kN)	D_y (m)	V (kN)	D (m)	S_a (g)	S_d (m)
X-direction	4788.09	0.0170	5291.54	0.0434	0.3343	0.0445
Y-direction	3768.94	0.0274	4896.79	0.0636	0.3086	0.0608
Z-direction	18205.76	0.0182	16603.35	0.0157	1.00	0.0323

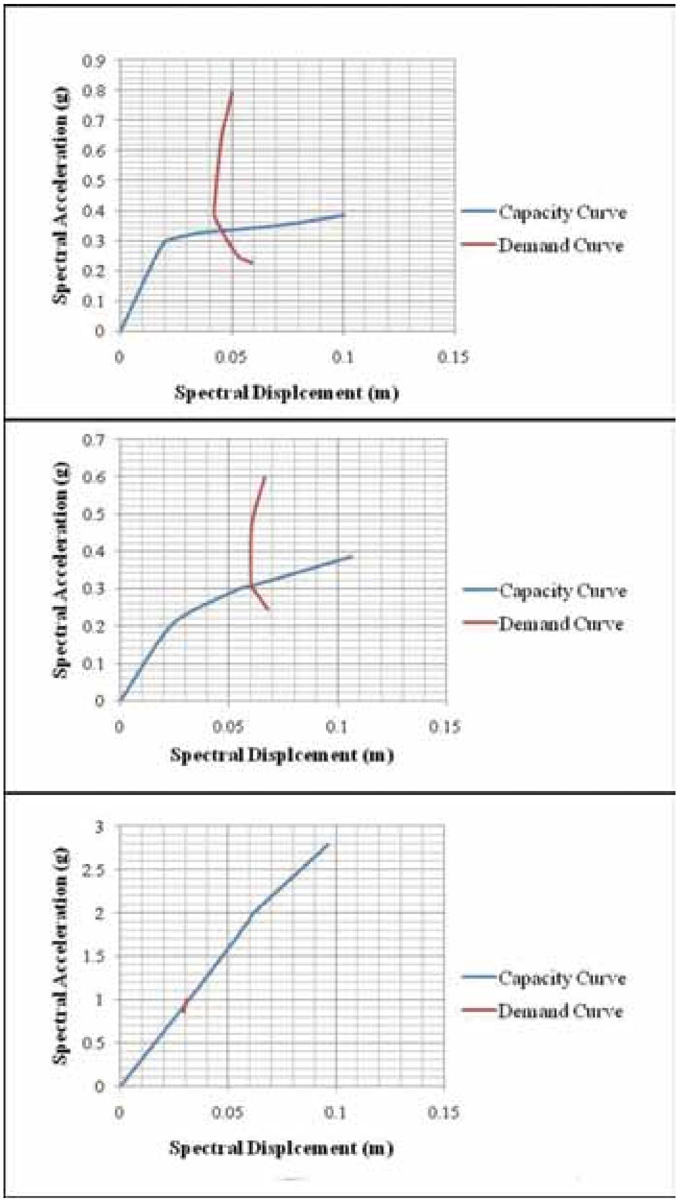

Figure 4. Capacity-demand curves for single barrel shell structure in X, Y and Z directions

Table 4. Maximum displacement in shell elements

Parameters	Maximum values	Pushover analysis		
		X-direction	Y-direction	Z-direction
Horizontal Displacement in X-Direction, U_x (m)	(+)	**0.1008**	0.0027	0.0051
	(-)	-0.0012	-0.0030	-0.0068
Horizontal Displacement in Y-Direction, U_y (m)	(+)	0.0082	**0.1150**	0.0264
	(-)	-0.0082	-0.0051	-0.0265
Vertical Displacement in Z-Direction, U_z (m)	(+)	0.0068	0.0274	**0.0795**
	(-)	-0.0196	-0.0458	-0.0168

2) Capacity Demand Curve and Performance Point

Capacity curves in X, Y and Z directions are obtained by pushover analysis. Then the capacity curves are transformed to the capacity spectrum curves. The Acceleration Displacement Response Spectra (ADRS) method is used for obtaining the demand curve. The procedure is adopted as mentioned in ATC 40. The capacity spectrum curve is superimposed on the demand spectrum curve and the intersection point is considered to be the performance point. Capacity Demand Curves are plotted between Spectral Displacement, S_d (m) and Spectral Acceleration S_a (g). The same are shown in Figure 4. Base shear and displacement corresponding performance point for different analysis cases are tabulated in Table 3. It is observed that the demand curve tend to intersect the capacity curve near the event point B, which means an elastic response and the security margin is greatly enhanced. Therefore, it can be said that the margin safety against collapse is high and there are sufficient strength and displacement reserves.

Table 5. Maximum membrane forces and bending moments in shell elements

Parameters	Maximum Values	Pushover Analysis		
		X-direction	Y-direction	Z-direction
Membrane Force in X-direction, F_{11} (kN/m)	(+)	2451.42	2262.71	3315.62
	(-)	-1649.04	-1649.96	**-5164.44**
Membrane Force in Y-direction, F_{22} (kN/m)	(+)	246.05	1651.72	**4800.25**
	(-)	-1674.53	-2898.06	-1016.91
In Plane Shear Force, F_{12} (kN/m)	(+)	595.90	668.84	1885.82
	(-)	-596.00	-658.03	**-1886.14**
Out of Plane Shear Force in X-direction, V_{13} (kN/m)	(+)	**389.74**	65.06	213.09
	(-)	-389.69	-73.98	-213.09
Out of Plane Shear Force in Y-direction, V_{23} (kN/m)	(+)	238.58	154.96	**388.62**
	(-)	-291.23	-153.22	-86.84
Bending Moment in X-Direction, M_{11} (kN-m/m)	(+)	13.59	9.74	29.99
	(-)	**-31.23**	-12.25	-16.07
Bending Moment in Y-Direction, M_{22} (kN-m/m)	(+)	18.03	95.67	113.08
	(-)	-44.17	**-134.73**	-65.57
Twisting Moment, M_{12} (kN-m/m)	(+)	19.29	49.77	32.22
	(-)	-19.29	**-51.66**	-32.22

Table 6. Maximum membrane stresses

Parameters	Maximum Values	Pushover Analysis		
		X-direction	Y-direction	Z-direction
Membrane Direct Stress in X-direction, S_{11} (kN/m^2)	(+)	4253.78	3787.06	3633.27
	(-)	-4444.69	-4080.45	**-10239.80**
Membrane Direct Stress in Y-direction, S_{22} (kN/m^2)	(+)	2845.76	6357.59	5736.56
	(-)	-4297.43	**-8702.55**	-6696.08
Membrane Shear Stress, S_{12} (kN/m^2)	(+)	2439.03	3427.77	**3466.25**
	(-)	-2438.94	-3461.11	-3466.04
Plate Transverse Shear Stress in X-direction, S_{13} (kN/m^2)	(+)	**1020.09**	169.01	599.98
	(-)	-1019.95	-191.08	-599.60
Plate Transverse Shear Stress in Y-direction, S_{23} (kN/m^2)	(+)	751.68	424.43	**1104.34**
	(-)	-901.95	-414.72	-243.84

3) Displacement in Shell

The maximum and minimum values of nodal displacements for pushover analysis cases are tabulated in Table 4. The maximum deflection is 0.10076 m in X direction, 0.11502 m in Y direction and 0.07955 m in Z direction.

4) Membrane Forces and Bending Moments

The internal membrane (in-plane) forces consists of two membrane normal resultant forces F_{11}, F_{22} and a membrane in-plane shear force F_{12} per unit length. The bending forces field consists of two bending moments M_{11}, M_{22} per unit length, a twisting Moment M_{12} of the shell cross-sections per unit length, and two transverse Out of Plane shear forces V_{13}, V_{23} per unit length. The X, Y and Z directions are denoted as 1, 2 and 3 in element resultants as per the software. The maximum and minimum values of the Membrane Forces per unit length for nonlinear static analysis cases are tabulated Table 5.

5) Stresses in Shell & Shell Layers

The basic shell element stresses are identified as S_{11}, S_{22}, S_{12}, S_{13}, and S_{23} for barrel shell structure. S_{11} and S_{22} denotes the Membrane Direct Stress in X and Y direction respectively. S_{12} denotes the Membrane shear stress. S_{13} and S_{23} denotes the Plate Transverse Shear Stress in X and Y direction respectively. The membrane stresses are tabulated in Table 6.

Table 7. Maximum membrane stresses

Parameters	Layers	Maximum Values	Pushover Analysis		
			X-direction	Y-direction	Z-direction
Maximum principal stress, S_{MAX}	Concrete Layer	(+)	3184.53	**4099.68**	2578.46
		(-)	353.62	1876.81	2551.44
	Top bar 1	(+)	**24797.55**	22920.80	19981.75
		(-)	7081.26	12053.83	15932.10
	Top bar 2	(+)	**24797.55**	22920.80	19981.75
		(-)	7081.26	12053.83	15932.10
	Bottom Bar 1	(+)	**26157.26**	21787.86	19797.66
		(-)	2242.60	3085.71	178.68
	Bottom Bar 2	(+)	**26157.26**	21787.86	19797.66
		(-)	2242.60	3085.71	178.68
Maximum principal stress, S_{MIN}	Concrete Layer	(+)	312.21	2019.82	1231.65
		(-)	2187.75	**3690.03**	2973.87
	Top bar 1	(+)	14860.83	16039.83	7525.29
		(-)	19995.95	**28554.05**	20504.00
	Top bar 2	(+)	14860.83	16039.83	7525.29
		(-)	19995.95	**28554.05**	20504.00
	Bottom Bar 1	(+)	435.92	4656.99	1698.42
		(-)	18999.17	**28839.42**	19823.69
	Bottom Bar 2	(+)	435.92	4656.99	1698.42
		(-)	18999.17	**28839.42**	19823.69

Maximum principle stress, S_{MAX} and minimum principle stress, S_{MIN} for whole thickness and different layers are tabulated in Table 7. The software provides the facility to find stresses for defined layers separately. Maximum and minimum principle stresses for shell, concrete layer, top reinforcement bar and bottom reinforcement bar are then compared with permissible stresses.

7. CONCLUSIONS

A three dimensional finite element analysis of Single barrel shell structure is performed using pushover analysis procedure to assess the nonlinear behavior of structures subjected to earthquake actions. Following conclusions are drawn from the analysis results.

1. The prominent modes in the X, Y and Z directions are Mode 1, 2 and 5 respectively. The modal analysis results show that mass participation in X-direction is 95.53 %, in Y-direction is 99.96% and in Z-direction is 37.57 %. As the mass participating factor in first and second mode is more than 90%, the dynamic response is dominated by first and second mode and it is expected that the pushover analysis yields realistic results.
2. Single Barrel Shell Structure is designed as per IS: 1893- Part I (2002), Criteria for earthquake resistant structure. The design spectrum used is of medium soil and the structure is considered in Zone-V. Design horizontal seismic coefficient is 0.09 and design seismic base shear obtained is 1494.30 kN. Base shear obtained by pushover analysis in X-direction is 4788.09 kN at yield point and 5291.54 kN at performance point. Base shear obtained by pushover analysis in Y-direction is 3768.94 kN at yield point and 4896.79 kN at performance point. Base shear obtained by pushover analysis in Z-direction is 18205.76 kN at yield point and 16603.35 kN at performance point. Displacement obtained by pushover analysis in X-direction is 0.017 m and 0.043 m at yield and performance point respectively. Displacement obtained by pushover analysis in Y-direction is 0.0274 m and 0.063 m at yield and performance point respectively. Permissible storey drift as per IS 1893:2002 due to lateral force shall not exceed 0.004

times the storey height. The permissible storey drift is 0.024 m for 6 m storey height. The storey drift is within limit for all the cases.

3. The permissible vertical deflection as per IS 456:2000 [22] is 0.144 m (span/250). The maximum absolute vertical displacement in shell element is equal to 0.0795 m. The deflection is within the permissible limit.

4. The maximum absolute membrane force in shell elements is 5164.44 kN/m and the maximum absolute out-of –plane shear force in shell elements is 389.74 kN/m. The out-of –plane shear forces are very less and hence negligible in comparison with membrane forces. The maximum absolute in-plane shear force in shell elements is 1886.14 kN/m and cannot be neglected. The maximum absolute bending moment is 134.73 kN-m/m and maximum absolute twisting moment is 51.66 kN-m/m. The bending and twisting moments are very low. This confirms the membrane resisting mechanism in shell structures. The absolute maximum membrane stress is 10239.8 kN/m^2. The maximum stress in concrete layer is 4099.677 kN/m^2 and 28839.42 kN/m^2 in steel layer. The permissible stresses in concrete and steel as per IS 456:2000 are 11150 kN/m^2 ($0.446*f_{ck}$) and 361050 kN/m^2 ($0.87*f_y$). The stresses in concrete and steel layers are within the permissible limit.

5. Indian standard codes lack in guidelines and specifications for nonlinear seismic analysis and design of shell structures to be adopted. An attempt has been made to predict the seismic response of RC barrel shell structure by pushover analysis.

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Ms. Raana Pathak holds a Bachelor degree in Civil Engineering; Masters degree in Structures from SGSITS, Indore; Pursuing her PhD at RGPV Bhopal. She is an Assistant professor at SGSITS, Indore. She has guided eleven PG dissertations. Her work interests mainly include earthquake engineering, structural dynamics, shell structures and fragility analysis.

Dr. Rakesh K. Khare holds a Bachelor degree in Civil Engineering; Masters degree in Stress and Vibrations Analysis of Machinery and Structures from Bhopal University; PhD from DAVV Indore. He has done One Semester Certificate Course at IIT Kanpur on Earthquake Resistant Design of Structures and six months Post Doctoral Research Training at University of Canterbury, Christchurch, New Zealeand. He has published research papers in various national and international journals and conferences. He has guided more than sixty PG dissertations. He is a Professor at SGSITS, Indore. His work interests mainly include earthquake engineering, structural dynamics, bridge engineering, shell structures and fragility analysis.