Effect of finite size joint correction on staging of grid type water tank

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INTRODUCTION

The analysis of staging of water tower is performed on the assumption that center line dimensions are to be used. However, the columns and braces always have some finite widths because of which the joint portion is reasonably rigid than middle portion. Hence, the static as well as dynamic parameters get altered. It was decided to work out approximate expressions for calculating lateral and torsional stiffness's considering finite size joint effect. It was proposed to check torsional vulnerability of grid type staging with and without joint effect. It was also proposed to study changes in buckling effect and dynamic properties of the grid type staging for water towers supported on four, nine and twelve columns. The results include simple, calculator friendly expressions for various stiffness's and comparison of above parameters. With minor modifications the results can be applicable for practically all grid type stagings with more number of columns.

Due to fixity, the deflection at mid span gets reduced, time period shortens. For seismic analysis, base shear increases, etc. Even though this particular aspect of fixity of rigid joints is well known and acceptable, it is hardly applied in practice. The same is very poorly reported in standard literature even for regular and common building structures. Ingle et al reported for building that if its effect is to be considered for design, the necessary provision has to be made during analysis itself [1]. Drona reported that applying finite size joint correction (FSJC) to whole structure further changes the parameters described above as compared to applying FSJC to braces only[2]. However, apart from these, not much work is reported on water towers and specially resting on grid types of stagings. Hence, it was decided to study effect of FSJC on water tower staging of grid type.

In case of water towers, the bottom beam is of size, usually much more than the braces provided to tie the columns. Furthermore as it is integrally cast with bottom slab, behaves as a T beam with stiffness at least two times that of equivalent rectangular section [3]. Hence, for all practical purposes as well as analysis it can be assumed to be infinitely rigid. Therefore, it was decided to study changes due to FSJC in columns and braces only. For grid type of staging with four, nine, twelve columns, lateral stiffness due to bending alone, lateral stiffness due to axial deformation and torsional stiffness is calculated and compared. It is observed that logic used for nine columns staging which consist of 3x3 columns square grid, can be suitably modified to other 4x4, 5x5, 6x6 square grid of columns staging. Similarly analysis for twelve columns staging (columns on two concentric circles) can be extended to grids supported on two, three or more concentric circles.

For grid type tank staging the c/c distance between columns generally varies from minimum of 3 m to maximum of 6 m; while column size varies from 400 mm to 750 mm. Thus width of column is in the range of 0.1 to 0.15 times span of the brace. Standard books on structural analysis such as Wilbur Norris, Wang etc. recommend that for approximate analysis of frames or continuous beams; clear span of length equivalent to 0.8 to 0.9 times c/c span of beam may be considered and analysis can be performed as simply supported [4,5]. IS 456:2000 states that; if the supports are wider than 1/12th of clear spans, for continuous or fixed spans, effective span shall be the clear span between the supports [6]. Macleod suggested formulae for rotational stiffness of steel frame including the effect of FSJC [7]. Based on his work Ingle et al suggested formulae which give directly the time period of tanks [8].

All the above studies do not include lateral stiffness due to axial deformation as well as torsional stiffness for tank staging in general and grid type staging in particular. The approximate analytical formulation for lateral stiffness due to bending, lateral stiffness due to axial deformation of columns and torsional stiffness considering center line dimensions is reported by Khandeshe and Ingle [9]. The formulae are reproduced herein Table 1 for ready reference.

Considering these basic formulae, suitable modifications for considering FSJC is presented below. Detailed derivation and application of the same to other similar grids can be found reported by Khandeshe [10]. The same grid type stagings are analyzed with a software considering semi rigid end zones and results are compared. Figures 1 to 3 give layouts of grid type stagings with four, nine and twelve columns with center to center as well as clear spans. Figure 4 depicts number of vertical panels for above stagings.

Lateral Stiffness due to bending

Considering ‘*C*’ as size of column and ‘d’ as depth of brace, L_C and h_C are clear spans of brace and clear height of panel respectively.

Defining parameters ‘β_c’ and ‘β_b’ as...

$$\beta_c = \frac{C}{h} \;,\; \beta_b = \frac{d}{L}$$

Nomenclature

A_C	Cross sectional area of smallest concrete column
C	Size/diameter of column
d	Overall depth of brace
E	Modulus of elasticity of concrete
H_T	Total height of staging
h	Height of typical panel of staging (center to center distance in brace) vertical plane)
I_O	Sum total of second moment areas of all columns
I_B	Second moment of area for beam or brace
I_{C1}, I_{C2}	Second moment of area for column external and internal respectively
L	Centre to center span of brace.
M_E	Seismic mass tank empty case
M_{BG}	Bending moment in ground brace
M_{BIM}	Maximum bending moment amongst intermediate braces
M_{CF}	Bending moment at top of foundation in column
M_{CT}	Bending moment at top of staging in column
N_c	Number of columns
N_P	Number of panels of staging in vertical plane
R	Radius of staging
$SF_{c/b}$	Ratio of total column stiffness to total brace stiffness in staging
T	Fundamental time period
T_E	Time period tank empty
T_F	Time period tank full
T_θ	Torsional period
V_b	Design base shear
W	Seismic weight of structure/tank
Δ_{Top}	Deflection at tank top
τ	Ratio of torsional period to lateral period
β_C	FSJC parameter for accounting column size
β_b	FSJC parameter for accounting brace size

Table 1. Stiffness formulae for various Stagings

Staging columns	Lateral Stiffness (Flexural)	Lateral Stiffness (Axial deformation of column)	Torsional Stiffness
Four	$\frac{12\,EI_O}{h^3\,N_p} \times \frac{1}{(1+1.50\,SF_{c/b})}$	$\frac{3\,E\,A_C\,L^2}{{N_P}^3\,h^3}$	$\frac{12\,E\,I\,N_C}{N_P\,h^3} \times \frac{R^2}{1+0.92\,SF_{C/B}}$
Nine	$\frac{12\,EI_O}{h^3\,N_p} \times \frac{1}{(1+1.50SF_{c/b})}$	$\frac{18.75\,E\,A_C\,L^2}{{N_P}^3\,h^3}$	$\frac{24\,E\,R^2}{N_P\,h^3\,SF_{C/B}} \times (I_{C1}+I_{C2})$
Twelve	$\frac{12\,EI_O}{h^3\,N_p} \times \frac{1}{(1+1.50\,SF_{c/b})}$	$\frac{33.5\,E\,A_C\,L}{N^{\;3}\,h^3}$	$\frac{12\,E\,I_{C1}R^2}{N_P\,h^3} \times \frac{7.6}{(\,SF_{C/B}+0.85\,)}$

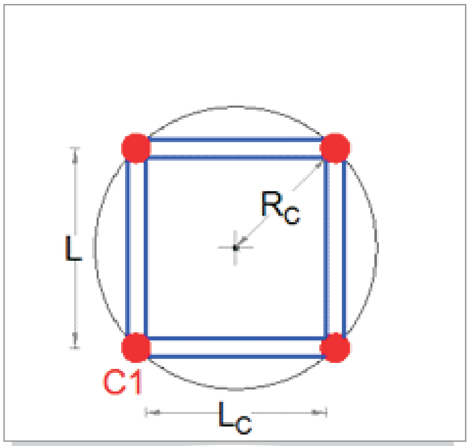


Figure 1. Four columns

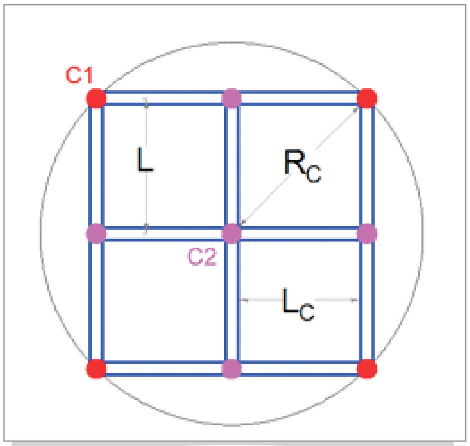


Figure 2. Nine columns

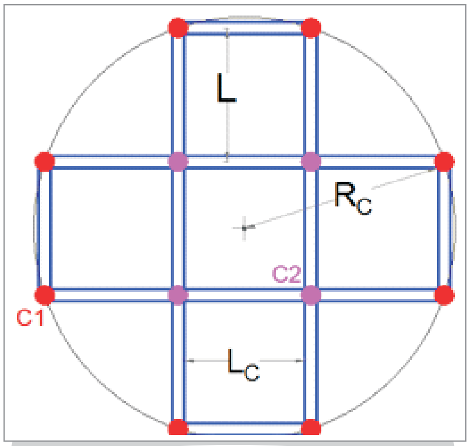


Figure 3. Twelve columns

Lateral stiffness due to bending alone can be written as:

$$K_{Lateral} = \frac{12\,E\,I_O}{h^3(1-\beta_c)^3 N_p} \times \frac{\beta_C}{(1+1.5\times SF_{c/b})} \quad \text{and}$$

$$SF_{c/b} = \frac{\sum_{i=1}^{N_C} \frac{I_C}{h\,(1-\beta c)}}{\sum_{i=1}^{N_b} \frac{I_b}{L(1-\beta_b)}} \qquad (1)$$

Eqn.(1) is applicable for all the three types of staging considered. It is found that due to application of FSJC to braces, the lateral stiffness due to bending increases.

Lateral stiffness due to axial deformation

Lateral stiffness due to axial deformation is primarily dependent on modulus of elasticity, second moment of area of columns, and height of panel. Cross section of braces, number of braces etc… have secondary effect on axial

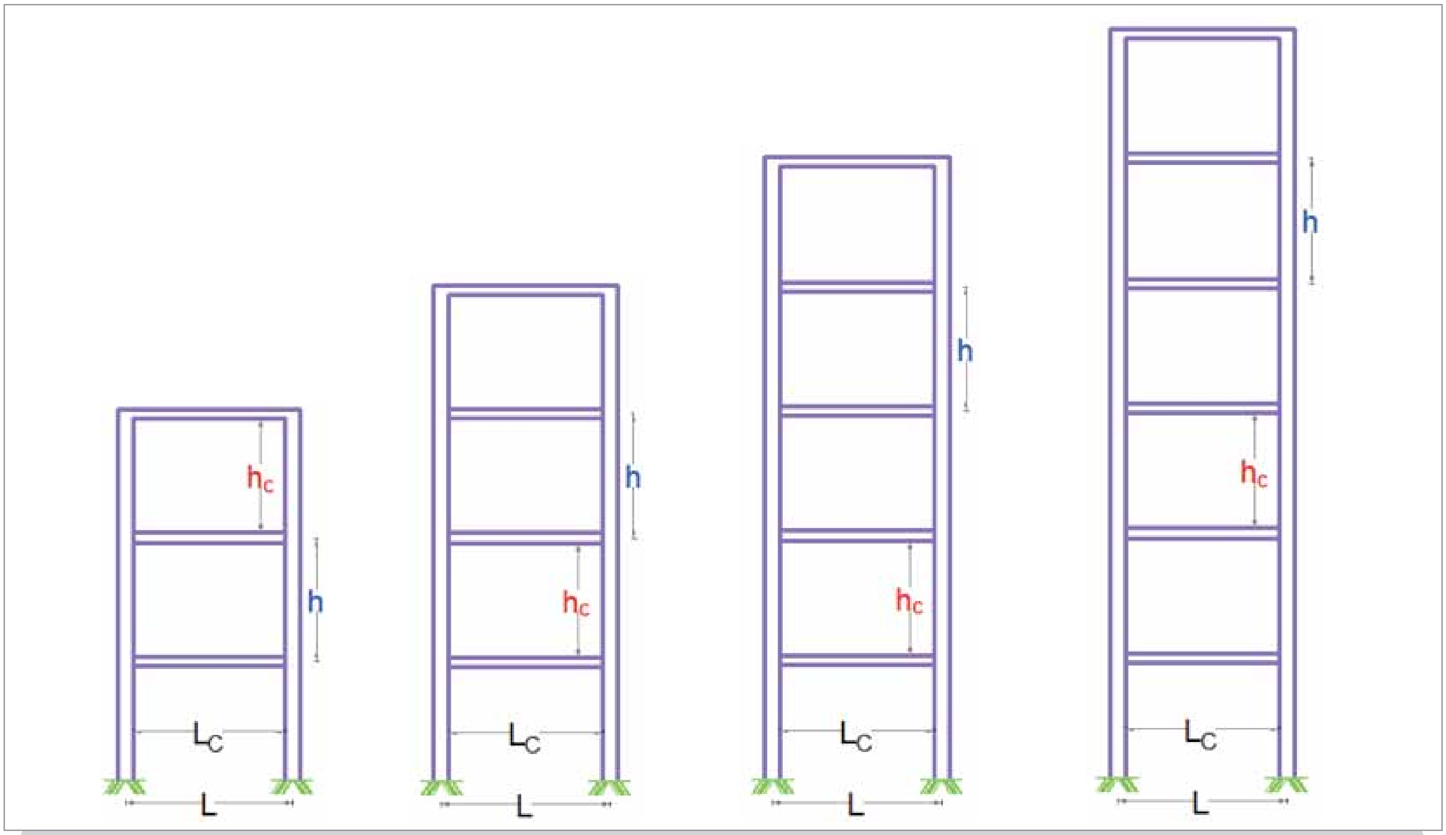


Figure 4. Three, four, five and six panels staging

deformation. Distance of column from center of gravity of staging (CG), which is related to span of brace have effect on stiffness of braces and consequently on axial deformation. However, this effect due to axial deformation is very small and generally in the range of 2 to 3%.

Lateral stiffness due to axial deformation for grid type tank staging with FSJC applied to the whole staging except top girder can be written as:

$$K_{Axial} = \frac{3\times E}{N_P{}^3\ h^3(1-\beta_c)^3} \times \sum_{i=1}^{N_{C1}} X_i{}^2\ A_i \qquad ...(2)$$

where 'X_i' is the distance measured between CG of staging to the concerned column measured along the direction of lateral force and 'A_i' is cross sectional area of column in the staging.

For four columns staging putting $X_i = 0.5\ L^*(1-\beta_b)$ in Eqn. (2)

$$K_{Axial} = \frac{3\ E\ A_C L^2(1-\beta_b)^2}{(N_P\ h\ (1-\beta_c))^3} \qquad ...(3)$$

For nine columns lateral stiffness due to axial deformation of columns is..

$$K_{Axial} = \frac{18.75\ E\ A_C\ L^2(1-\beta_b)^2}{(N_P\ h\ (1-\beta_c))^3} \qquad ...(4)$$

Similarly for twelve columns staging:

$$K_{Axial} = \frac{33.5\ E\ A_c\ L^2(1-\beta_b)^2}{(N_P\ h\ (1-\beta_c))^3} \qquad ...(5)$$

It is observed that with introduction of FSJC to braces, lateral stiffness due to axial deformation decreases.

Torsional stiffness

Torsional stiffness of frame of staging is dependent on number of columns in the staging, cross section of columns, distance of each column or group of columns from CG, height of panel, number of panels etc. Considering FSJC, contribution due to distance of column from CG gets affected. All other parameters have little effect due to FSJC.

Torsional stiffness for four columns grid type staging for general case is:

$$K_\theta = \frac{12\ E\ I_C\ N_C}{N_P\ h^3} \times \frac{R^2}{1+0.92\times SF_{C/B}} \qquad ...(6)$$

For four columns on periphery of circle, $R^2 = 0.5\ L^2$ and considering FSJC Eqn. (6) can be rewritten as:

$$K_\theta = \frac{12\ E\ I_C\ N_C}{N_P\ h^3\ (1-\beta_c)^3} \times \frac{R^2(1-0.7071\beta_b)^2}{1+0.92\times SF_{C/B}} \qquad ...(7)$$

For staging with nine columns considering FSJC

$$K_\theta = \frac{24\ E\ R^2(1-\beta_b)^2}{N_P\ h^3(1-\beta_C)^3\ SF_{C/B}} \times (I_{C1}+I_{C2}) \qquad ...(8)$$

For staging with twelve columns

$$K_\theta = \frac{12\ E\ I_{C1}R_C{}^2}{N_P\ h^3(1-\beta_C)} \times \frac{7.6}{(\ SF_{C/B}+0.85)} \qquad ...(9)$$

where, R_c= R - column dia

Degree of fixity of column brace junction

Degree of fixity of joint decides the proportion in which forces at joint get distributed amongst the structural elements. Fixity of a column beam joint is basically dependent on stiffness of members, angle of inclination of members meeting at the joint, percentage of main reinforcement in respective members, amount and placement of transverse reinforcement, grade of concrete etc... In spite of lot of research, there is no quantitative measure to calculate exact degree of fixity at joint. Concrete being brittle material, concrete codes such as IS 456:2000 generally prescribe limit for redistribution of moments, not more than 30%[6].While 0% fixity implies center line dimensions; 100% fixity means full fixity. Both these extreme cases are next to impossible to achieve in practice for water towers. Hence, it was decided to provide 0%, 50%, 67% and 100% fixity to column brace joints and study its effect on various stiffness's to finalize amount of fixity for further work.

Table 2 presents salient details of water tank stagings with four, nine and twelve columns, which are analyzed in present studies.

Table 2. Salient details of tanks analyzed

Parameter	Four columns	Nine columns	Twelve columns
Concrete Grade f_{ck}	M 30	M 30	M 30
$E_c = 5000.\sqrt{f_{ck}}$ Mpa	27386	27386	27386
Staging Radius Rs m.	3.43	5.38	5.53
Brace Span L (m)	4.85	3.80	3.50
Braces/Panel	4	12	16
Brace Size (m)	0.25 x 0.40	0.25 x 0.40	0.25 x 0.40
Panel Height h (m)	4	4	4
Column Size (m)	0.45	4 Nos - 0.50 5 Nos- 0.55	8 Nos - 0.45 4 Nos- 0.50
Number of Panels, Np	3,4.5.6	3,4.5.6	3,4.5.6

Out of this a typical case of staging of four columns, four panels is considered for deciding degree of fixity of joints. It is analyzed with a software for four different percentages of fixity and results are presented in Table 3.

Table 3. Four columns four panels staging with different degrees of fixity

Stiffness	Degree of Fixity			
	0 %	50 %	67 %	100 %
Lateral Stiffness kN/m (Flexure) , Diff %	2959 0	3410 15.2	3537 19.5	3803 28.5
Lateral Stiffness, kN/m (Axial deformation), Diff %	81519 0	80874 -1	80942 -1	80712 -1
Torsional Stiffness, kN-m Diff %	50000 0	55556 11	55556 11	58824 18

From Table 3 it can be seen that lateral stiffness changes from 0% to 28 % as degree of fixity varies from 0% to 100 %. For torsional stiffness the corresponding values range from 0% to 18% respectively. There is not much of a difference in lateral stiffness due to axial deformation of columns because of amount of fixity. Barring two extreme cases of 0% and 100 % fixity, as water tower is an important structure with reasonable quality control on workmanship, it seems prudent to use 67% fixity. This is in tune with IS 11682, Draft Code which specifies values in the range of 0.5 to 1 [11].

For verifying accuracy of the stiffness formulae as per Eqns. (1) to(9), FSJC as shown in Figure 5 is applied to staging consisting of four, nine and twelve columns, and finite element analysis (FEA) by a software is performed for the tanks data presented in Table 2.

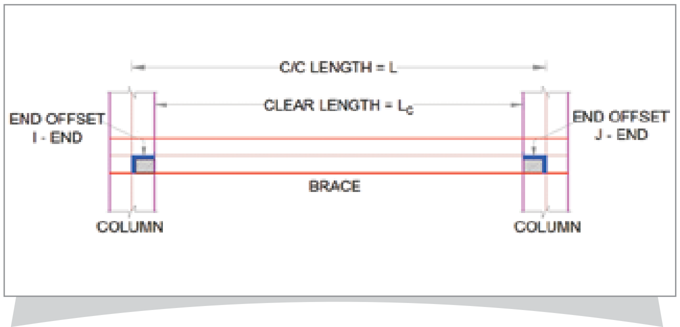


Figure 5. Typical column brace junction - End offsets

Lateral stiffness due to flexure and axial deformation and torsional stiffness are worked from above equations and compared with FEA values as shown in Table 4.

From Table 4, it is seen that for four column stagings, Eqn. 1 for lateral stiffness due to flexure, give values within 1% to 10% of FEA values for four to six panels. Expressions for lateral stiffness due to axial deformation (Eqn. 3) yield values within 1% to 8% range as compared to FEA. Torsional stiffness (Eqn. 7) varies in between 1% to 5% of FEA values for four to six panels. Lateral as well as torsional stiffness increases as number of panels decrease.

It is observed that if the lateral stiffness due to axial deformation of columns is not considered in calculating overall stiffness, it is over estimated to the tune of 2% to 8% as number of panels increase from three to six. For less number of panels, the frame being more stiff axial deformation of columns do not contribute much to the overall stiffness. When number of panels increase, the frame becomes flexible and hence, contribution of axial deformation is more.

Table 4. Four columns staging with three, four, five and six panels

Panels	Lateral Stiffness Flexure, kN/m			Lateral Stiffness Axial Deformation, N/m			Torsional Stiffness, kN-m/m		
	FEA	Eqn.1	Diff, %	FEA	Eqn.3	Diff, %	FEA	Eqn.7	Diff, %
Three	5323	4516	15	198965	200816	1	83333	71985	-14
Four	3410	3387	-1	80874	84720	5	55556	53990	-3
Five	2825	2709	4	40343	43377	7	45455	43192	-5
Six	2509	2258	10	23173	25103	8	35174	35993	-1

Table 5. Nine columns staging with three, four, five and six panels

Panels	Lateral Stiffness Flexure, kN/m			Lateral Stiffness Axial Deformation, kN/m			Torsional Stiffness, kN-m/m		
	FEA	Eqn.1	Diff, %	FEA	Eqn.4	Diff, %	FEA	Eqn.8	Diff, %
Three	23027	17756	-22	862895	871598	1	500000	447286	-10
Four	13140	13317	1	350195	367706	5	333334	335464	1
Five	10058	10654	6	170326	188265	10	250000	268371	7
Six	8462	8878	5	102552	105890	3	200000	223643	12

Table 6. Twelve columns staging with three, four, five and six panels

Panels	Lateral Stiffness Flexure , kN/m			Lateral Stiffness Axial Deformation, kN/m			Torsional Stiffness, kN-m/m		
	FEA	Eqn.1	Diff, %	FEA	Eqn.5	Diff, %	FEA	Eqn.9	Diff, %
Three	25022	22287	-11	988944	1077500	9	500000	450061	-9
Four	15884	16715	5	410920	454570	10	333333	337546	1
Five	12095	13372	11	207848	232740	12	250000	270037	8
Six	10908	11144	2	120712	134687	11	250000	225031	-10

From Table 5 it can be seen that for lateral stiffness due to flexure, approximate expressions give values within 1% to 6% of FEA values for four to six panels. Equations for lateral stiffness due to axial deformation (Eqn. 4) give values within 1% to 10% range as compared to FEA. Torsional stiffness (Eqn. 8) varies in between 1% to 12% of FEA values for all panels ranging from three to six.

From Table 6 it is seen that, for lateral stiffness due to flexure, approximate equations give values within 2% to 10% of FEA values for three to six panels. Expressions for lateral stiffness due to axial deformation (Eqn. 5) yield values within 1% to 11% range as compared to FEA values. Torsional stiffness (Eqn. 9) varies between 2% to 10% of FEA values for all panels ranging from three to six.

Comparison of fundamental time period

Even though the stiffness may range between 2% to 18% of FEA, the fundamental time period is directly proportional to square root of mass of the structure and inversely proportional to stiffness. Hence, it is decided to compare fundamental time period in tank full and tank empty cases using the stiffness as calculated in Tables 4 to 6 above with time period calculated by FEA. The results are tabulated.

It is seen from Table 7 that, fundamental period calculated from approximate expressions as above, is in good agreement with the period calculated from FEA. While the difference is from 4% to 7% for four to six panels, it is in the range of 1% for staging with three panels. These results are valid for both tank full and empty cases.

It can be seen from Table 8 that fundamental period calculated from approximate expressions is in good agreement with the period from FEA. While the difference is from 3 to 7% for four to six panels, it is in the range of 2% for staging with three panels. These results hold good for both tank full and empty cases.

Table 7. Four columns staging with three to six panels

Panels	Tank Case	Mass of Tank (Kg)	Stiffness* of frame (kN/m)	T (Eqn.1,3)	T, FEA	Diff, %
Three	Full Empty	215748 100336	4896	1.32 0.90	1.31 0.90	0.7 0.0
Four	Full Empty	220900 105487	3604	1.56 1.07	1.68 1.14	-7.1 -7.0
Five	Full Empty	224557 109144	2816	1.77 1.24	1.9 1.34	-6.7 -7.5
Six	Full Empty	228213 112801	2036	2.10 1.48	2.19 1.55	-4.1 -4.5

* $\frac{1}{K_{frame}} = \frac{1}{K_{lateral}} + \frac{1}{K_{axial}}$ where K_{lateral} and K_{axial} are from Table 4

Table 8. Nine columns staging with three to six panels

Panels	Tank Case	Mass of Tank , (Kg)	Stiffness* of frame (kN/m)	T (Eqn.1,4)	T, FEA	Diff, %
Three	Full Empty	556895 219487	20113	1.05 0.66	1.06 0.68	-0.9 -1.5
Four	Full Empty	565807 228399	14307	1.25 0.79	1.3 0.83	-3.8 -4.8
Five	Full Empty	574719 237312	11203	1.42 0.91	1.52 0.97	-6.5 -6.1
Six	Full Empty	583632 246224	9100	1.59 1.03	1.72 1.16	-7.5 -11.2

* $\frac{1}{K_{frame}} = \frac{1}{K_{lateral}} + \frac{1}{K_{axial}}$ where K_{lateral} and K_{axial} are from Table 5

Table 9. Twelve columns staging with three to six panels

Panels	Tank Case	Mass of Tank (Kg)	Stiffness* of frame (kN/m)	T (Eqn.1,5)	T, FEA	Diff, %
Three	Full Empty	661980 168840	26823	0.99 0.50	1.08 0.53	-9.0 -6.0
Four	Full Empty	675261 182121	14199	1.37 0.71	1.36 0.67	0.7 5.8
Five	Full Empty	686959 193819	11159	1.56 0.83	1.54 0.79	1.3 5.0
Six	Full Empty	698657 205517	8282	1.82 0.99	1.73 0.91	5.2 8.7

* $\frac{1}{K_{frame}} = \frac{1}{K_{lateral}} + \frac{1}{K_{axial}}$ where K_{lateral} and K_{axial} are from Table 6

It can be seen from Table 9 that, fundamental period calculated from approximate expressions is in good agreement with the period from FEA. While the difference is from 1% to 8% for four to six panels, it is in the range of 9% for staging with three panels. These results hold good for both tank full and empty cases.

Comparison of Time Period with and without FSJC

Consideration of FSJC implies some amount of fixity of joints. Hence, the structure becomes rigid as compared to original. Hence, fundamental time period gets reduced. A comparison of time period is presented for four, nine and twelve columns staging.

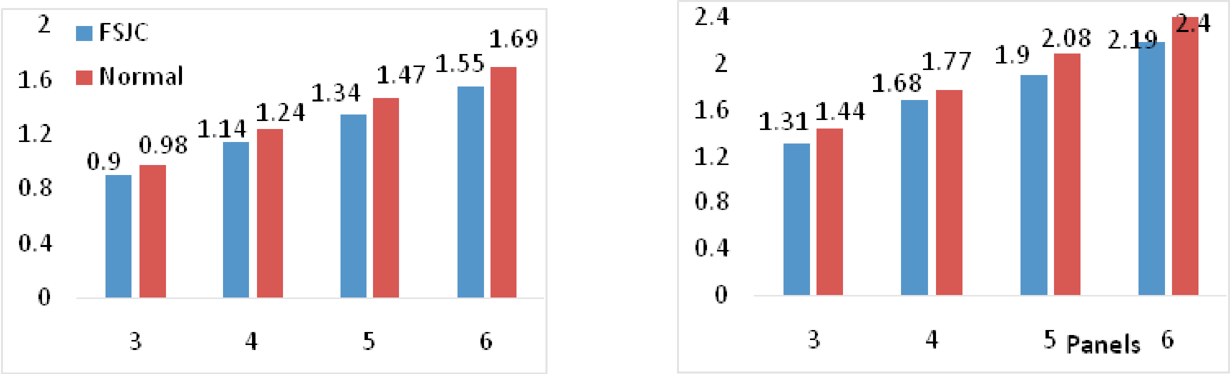
From Figure 6a it can be seen that with application of FSJC, time period for four column staging reduces by 10% for tank empty case. This reduction is 6% to 10% for tank full case (Figure 6b).

From Figure 7a it can be seen that, with application of FSJC, time period for nine column staging reduces by 9% to 14% for tank empty case. This reduction is 12% to 16% for tank full case as seen in Figure 7b.

From Figure 8a it is observed that with application of FSJC, time period for twelve column staging reduces by 8% to 9% for tank empty case. As per Figure 8b, this reduction is 10% to 13% for tank full case.

Comparison of Torsional Vulnerability

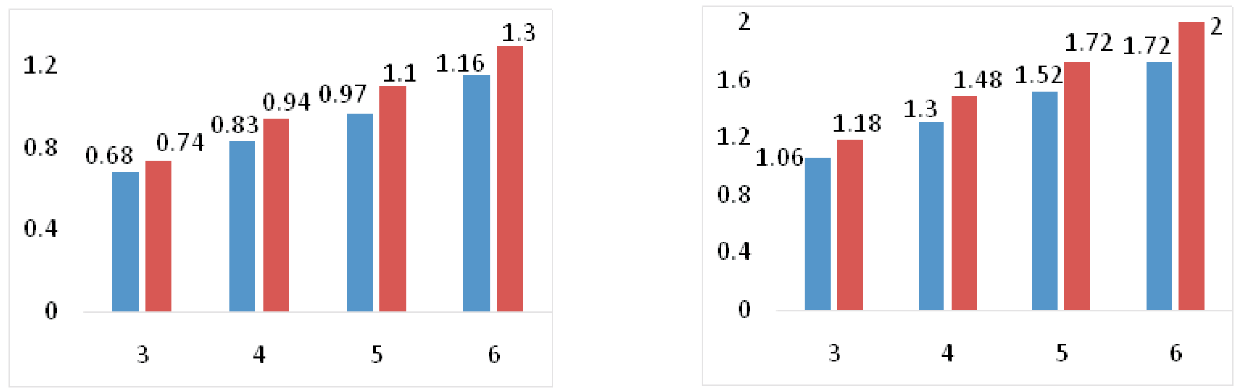
Elevated water tanks, because of their axisymmetric geometry and uniform mass distribution generally do not have eccentricity between center of mass and center of rigidity. Hence, the structure should not experience



6a: Time Period – Tank Empty

6b: Time Period – Tank Full

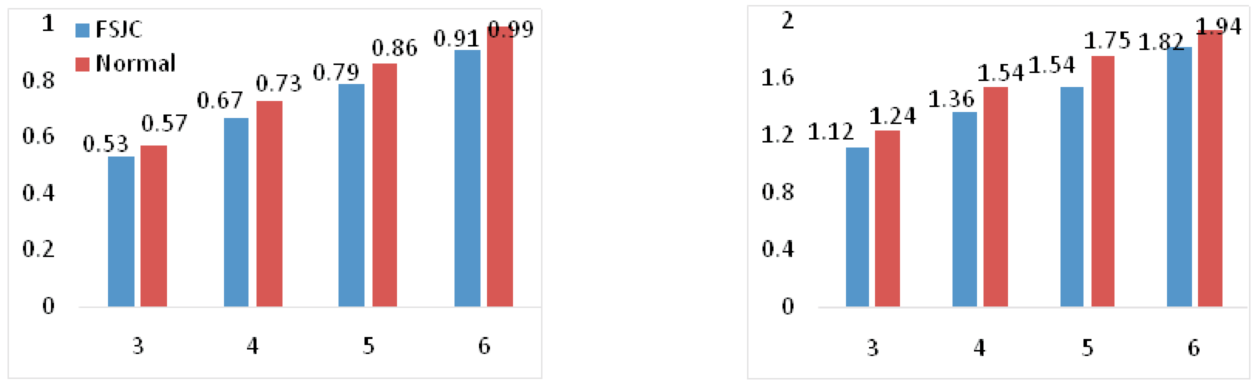
Figure 6. Four columns staging



7a: Time Period – Tank Empty

7b: Time Period – Tank Full

Figure 7. Nine columns staging



8a: Time Period Tank Empty

8b: Time Period Tank Full

Figure 8. Twelve columns staging

torsion because of it. However, asymmetrical placement of ladders, concrete staircase, pipe assemblies, non-uniformity of construction may introduce small eccentricity. It is also established that such small eccentricity can cause considerably amplified rotational response in structure if the ratio of torsional to lateral time period ratio 'τ'is near to 1. This is because, the structure is expected to enter the inelastic range. Dutta et al reported that tanks within an approximate critical range of $0.7 < \tau < 1.25$ may have amplified displacement due to coupled lateral-torsional motion. [12]. This can cause increasing localized damage in the yielded structural elements due to strength deteriorating properties of concrete under cyclic loading during an earthquake. Dutta has analyzed tank staging with columns on periphery of one and two concentric circles [13]. Hence, it is proposed to assess torsional vulnerability of grid type staging with and without FSJC.

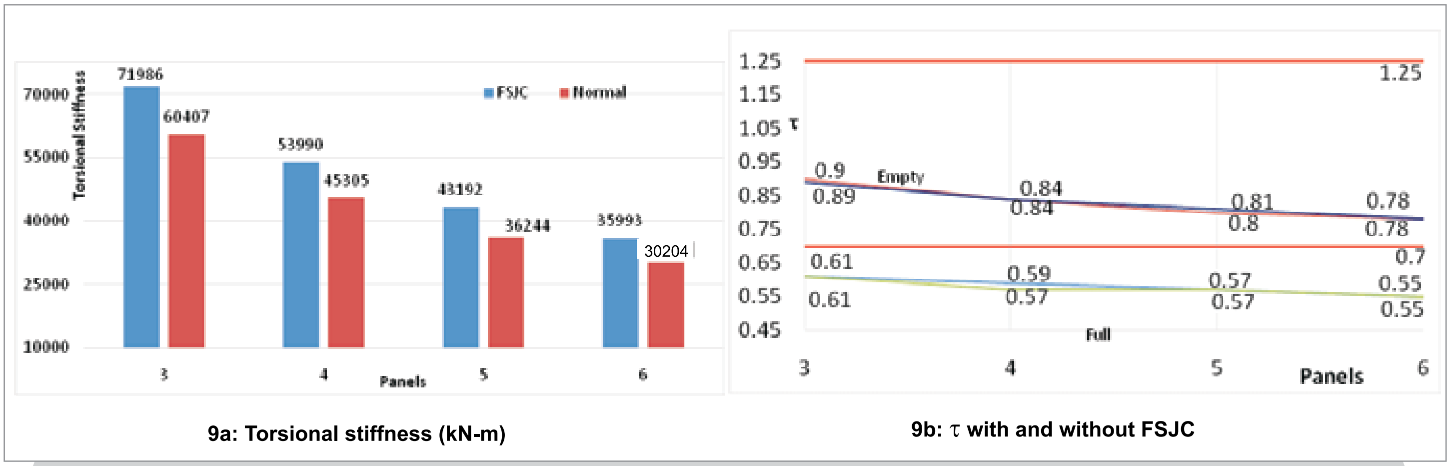
In torsional mode, the shear stress between the tank walls and the water is conceived to be inadequate to mobilize significant amount of water to vibrate with the tank in impulsive torsional mode. Hence, irrespective of tank empty or tank full, the mass moment of inertia for torsional vibrations shall be only that of tank structure. Hence, torsional period of tank is:

$$T_\theta = \sqrt{\frac{I}{K_\theta}} \qquad ...(10)$$

where, I is mass moment of inertia = $M_E\, R_g^{\,2}$

M_E =Seismic Mass for Tank empty case and

R_g is radius of gyration = R which is radius of container for no accidental eccentricity.



9a: Torsional stiffness (kN-m)

9b: τ with and without FSJC

Figure 9. Four columns staging

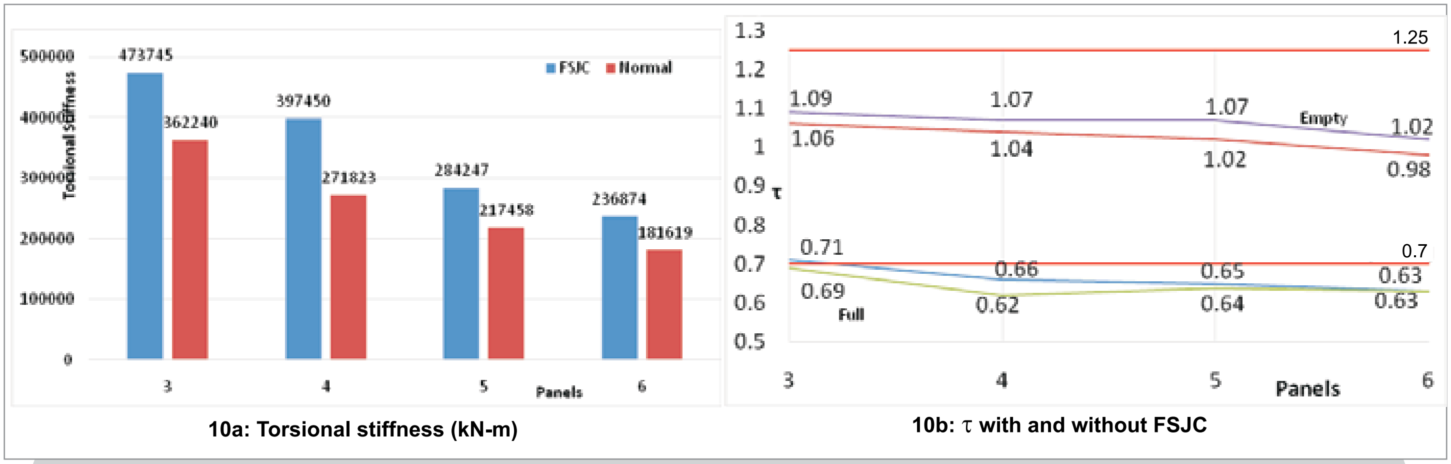
For the values of torsional stiffness obtained from approximate expressions given in Eqns. 6 to 9, torsional period is calculated for tank empty case and it is compared with lateral period as calculated previously for both tank full (T_{XF}) and tank empty (T_{XE}) cases. The same procedure is repeated for three to six panels staging with and without application of FSJC and the ratio; torsional period to lateral period (τ) is compared.

From Figure 9a it can be seen that torsional stiffness increases 18% to 20% when FSJC is considered. As number of panels increase from three to six, torsional stiffness in both cases reduces up to 50%. Torsional period increases as number of panels increase from three to six. Similarly, with application of FSJC torsional period reduces by 10%.

The ratio of torsional to lateral period is more or less not affected by application of FSJC. This is evident from Figure 9b as curves with and without FSJC are practically same. For tank full case 'τ'is between 0.55 to 0.61; which is well outside the critical range of 0.7 to 1.25 with and without considering FSJC.

However, for tank empty case, 'τ' lies between 0.78 to 0.90, for both cases and all number of panels. This is the vulnerable range as far as torsional response of structure is considered.

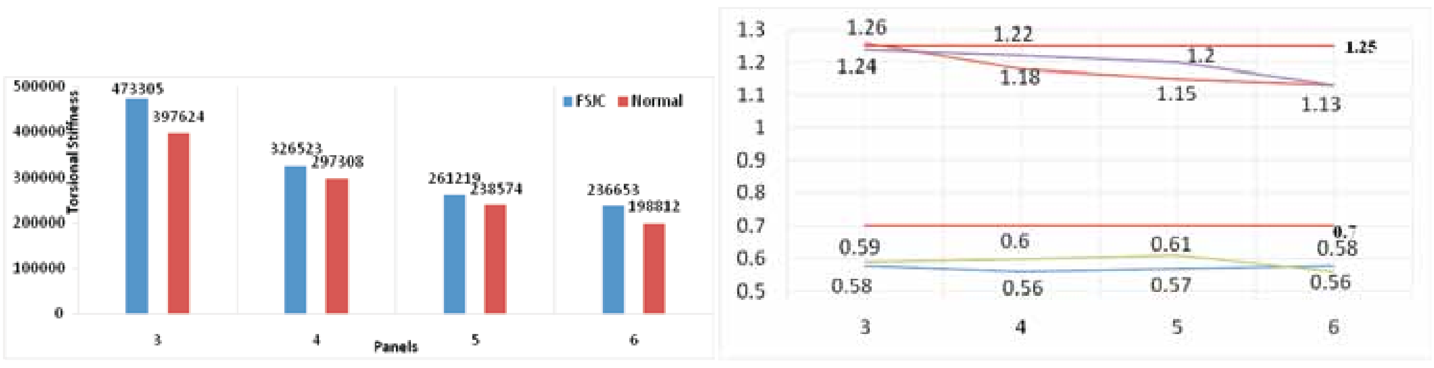
From Figure 10a it is seen that, torsional stiffness increases 20% to 25% when FSJC is considered. As number of panels increase from three to six, torsional stiffness in both cases reduces up to 50% i.e. increase in torsional stiffness for



10a: Torsional stiffness (kN-m)

10b: τ with and without FSJC

Figure 10. Nine columns staging



11a: Torsional stiffness (kN-m)

11**b: τ with and without FSJC**

Figure 11. Twelve columns staging

squat type stagings. Torsional period increases as number of panels increase from three to six. At the same time with application of FSJC torsional period almost reduces by 10%.

The ratio of torsional to lateral period ‘τ’ is more or less not affected by application of FSJC. For tank full case ‘τ’ is between 0.63 to 0.7; which is well outside the critical range of 0.7 to 1.25 with and without considering FSJC. (Figure 10b)

However for tank empty case, ‘τ’lies in between 0.98 to 1.12, with and without FSJC and three to six number of panels. Hence, for tank empty case torsional response of structure seems to be more predominant as compared to lateral response.

From Figure 11a, it is clear that, torsional stiffness increases 10% to 15% when FSJC is considered. As number of panels increase from three to six, torsional stiffness in both cases reduces up to 50%. Torsional period increases as number of panels increase from 3 to 6. At the same time with application of FSJC torsional period reduces by 10%.

The ratio of torsional to lateral period ‘τ’ is more or less not affected by application of FSJC. For tank full case ‘τ’ is between 0.55 to 0.6; which is well outside the critical range of 0.7 to 1.25 with and without considering FSJC. (Figure 11b)

However, for tank empty case, ‘τ’ lies in between 1.12 to 1.18, when FSJC is not considered. When FSJC is considered for three to five panels, ‘τ’ lies outside the critical range. Only when number of panels is six, it is in the critical range. Hence for tank empty case torsional response of structure shall be closely monitored especially when panels are six or more.

Comparison of Buckling Load

Stiffness is significantly affected by nature of axial force in the member. While stiffness increases when member is subjected to tensile forces, for compressive forces on the verge of buckling, lateral stiffness reduces significantly. Ghali reports that this reduction can be even up to 25% of original value if the buckling factor (or the ratio of Euler load to the actual load) is less than 2. [14]. Buckling can be defined as change of state of equilibrium from stable to neutral. Hence, even though the compressive stresses are well within safe limits, the structure may not be able to maintain its original form. For a tall, slender structure like water tank, with majority of mass concentrated at top, this type of failure is most likely to precede before material failure. The effective length as calculated by Euler's formulae for a fixed- fixed case is 0.5 times height of column. This increases even up to infinity for a sway frame such as water tower when end conditions reach hinged-hinged, as per Figure 27 of IS 456 [6].

As introduction of FSJC, brings about some amount of fixity to joint and thereby to connected columns, it is decided to perform buckling analysis for the same frames analyzed above and compare the buckling factors with and without application of FSJC. Buckling analysis for gravity loads (dead+ live) with stiffness considered at the end of nonlinear case,is performed in a software for the tanks mentioned in Table 2. For the loads specified in Tables 7, 8 and 9 for four, nine and twelve columns staging respectively, buckling factor which is the ratio of load in first buckling mode to gravity load is calculated and presented in Table 10.

Table 10. Comparison of Buckling Factor: Four, nine and twelve columns staging

Panels	Four column			Nine columns			Twelve column		
	without FSJC	with FSJC	Diff %	without FSJC	with FSJC	Diff %	without FSJC	with FSJC	Diff %
Three	20.9	24.1	15.3	30.3	37.1	22.4	27.7	33.9	22.4
Four	17.5	19.6	12.0	25.9	32.4	25.1	23.9	29.5	23.4
Five	15.8	18.6	17.7	23.3	29.6	27.1	22.7	28.5	25.5
Six	14.6	17.3	18.5	20.8	26.2	26.0	21.7	27.3	26.3

From Table 10 it is seen that when FSJC is applied, for four columns staging, buckling factor increases by about 12% to 18%. This increase in buckling factor is from 22% to 27% for nine and twelve columns staging respectively.

As number of panels increase from three to six, buckling factor decreases. This decrease is about 30% for four columns staging while it is up to 50% and 20% for nine and twelve columns staging respectively. This behavior is rather expected because as number of panels increase, flexibility of staging increases and consequently structure is more vulnerable for buckling.

Comparison of Dynamic and other Force Parameters

Along with buckling, top deflection and base shear also get affected by stiffness of structural elements. Hence, it was decided to assess the effect of FSJC on the above properties for stagings with four, nine and twelve columns. Response spectrum analysis is performed for the tank full case with and without FSJC, considering seismic zone IV and Type I soil as per IS 1893:2002 [15]. For the tank staging data presented in Table 2, importance factor considered is 1.5 and 5% constant damping, for all modes. Single mass model is considered for analysis.

While comparing Complete Quadratic Combination (CQC) and Square Root of Sum of Squares (SRSS) methods of modal superposition for performing response spectrum analysis, Wilson observed that the application of the CQC method allows the sum of the base shears in the direction of the external motion to be added directly [16]. In addition, the sum of the base shears, normal to the external motion, tends to cancel. The ability of the CQC method to recognize the relative sign of the terms in the modal response is the key to the elimination of errors in the SRSS method.For nine columns staging with three panels it was decided to combine the modes by both CQC and SRSS method and results were compared. While SRSS method gave base shear as 15.62 kN; CQC method resulted in a value of 19.82 kN. Hence, the further calculations were performed by CQC method.

Along with top deflection (mm) and base shear (kN), the force parameters which govern the design of water tank in general and staging in particular, are bending moment (kN-m) in column at footing top (M_{CB}), bending moment in column at staging top (M_{CT}), bending moment in ground brace (M_{BG}), and maximum bending moment in other than ground brace (M_{BIM}). These forces are evaluated for the critical condition and results are compared in Tables 11 to 13 for four, nine and twelve column stagings respectively.

From Table 11, it is seen that deflection at top of staging decreases up to 8% with introduction of FSJC for all the panels. While base shear increases by about 8% to 9%, bending moment in column at footing top increases by 2% to 4% by considering fixity of joints.

Table 11. Comparison-Deflection, base Shear and moments: Four columns staging

	Three Panels			Four Panels			Five Panels			Six Panels		
	C/c	FSJC	Diff	C/c	FSJC	Diff	C/c	FSJC	Diff	C/c	FSJC	Diff
Δ_{Top}	8.7	8.0	-8	11.1	10.3	7.2	13.2	12.1	8.3	15	14.0	7.9
V_b	40.3	43.4	8	33.2	36.1	8.8	29.6	31.9	7.8	27	29.6	8.4
M_{CB}	32.0	33.3	4	25.5	26.0	2.0	23.3	23.8	2.2	21	22.0	2.3
M_{CT}	103	116	13	98.6	112	13	96.3	109	14	94	108	12
M_{BG}	34.1	38.0	11	30.5	34.3	13	27.6	30.7	11	25	28.9	13
M_{BIM}	39.3	43.4	10	34.2	37.8	11	31.8	34.7	9.2	29	32.3	9.2

Table 12. Comparison-Deflection, base Shear and moments: Nine columns staging

	Three Panels			Four Panels			Five Panels			Six Panels		
	C/c	FSJC	Diff	C/C	FSJC	Diff	C/c	FSJC	Diff	C/c	FSJC	Diff
Δ_{Top}	7.4	6.7	-9.5	9.2	8.2	-11	10.9	9.7	-11	12.	11.4	-11
V_b	123	137	11.4	104	116	12	91.8	103	13	82.	91.9	12
M_{CB} C1	34.4	35.4	3.0	28.8	29.6	3.1	25.8	26.5	2.7	17	17.9	5.3
C2	50.0	53.2	6.4	43.9	45.6	3.6	39.2	40.7	3.8	25	27.0	4.2
M_{CT} C1	65.0	68.2	4.9	59.8	62.2	4.0	56.7	59.3	4.4	54	56.7	3.5
C2	47.1	51.1	8.5	45.2	49.6	9.7	43.3	47.5	9.7	40	44.2	8.6
M_{BG}	35.1	41.9	19.3	30.2	36.1	20	27.2	32.5	20	25	30.2	18
M_{BIM}	37.6	44.4	18.1	34.5	39.8	15	32.1	37.0	15	31.7	36.3	15

Table 13. Comparison-Deflection, base Shear and moments: Twelve columns staging

	Three Panels			Four Panels			Five Panels			Six Panels		
	C/c	FSJC	Diff	C/c	FSJC	Diff	C/c	FSJC	Diff	C/c	FSJC	Diff
Δ_{Top}	7.8	7.0	-12	9.7	8.6	-11	11.0	9.8	-11	12	11.0	-11
V_b	156	174	11.5	131	145	11	119	133	12	108	122	12
M_{CB} C1	29.9	31.1	4.0	25.2	26.1	3.6	22.6	23.5	4.0	20	21.2	2.4
C2	46.7	48.8	4.3	39.6	41.2	4.1	35.6	37.4	5.1	32	34.1	5.0
M_{CT} C1	43.4	49.8	14.8	28.3	31.7	12	33.4	38.4	15	31	36.2	15.7
C2	47.3	52.3	10.7	44.5	49.6	11	31.1	33.4	7	27	30.2	8.9
M_{BG}	35.8	41.5	15.9	30.8	35.8	16	28.3	32.9	16	25	30.2	19
M_{BIM}	39.4	45.5	13.4	34.7	39.1	13	31.9	35.2	10.4	29.1	32.0	10

Maximum increase in bending moment at top of staging is up to 13%. Increase in bending moment in ground and intermediate braces is of the order of 9% to 12%.

It is interesting to note that values of all force parameters decrease as number of panels increase from three to six. This is due to the fact that with increase in panels, the structure becomes more flexible and attracts less lateral forces.

For nine columns staging as per Table 12, deflection at top of staging decreases up to 9% to 11% with introduction of FSJC for three to six panels.

Increase in base shear is about 11% to 13%. Bending moment in external column at footing top increases by 3% to 4% with introduction of FSJC, the effect is 5% to 6% for central columns. Maximum increase in bending moment at top of staging is up to 5% for external columns, while it is up to 9% for middle and central columns.

Increase in bending moment in ground brace is of the order of 18% to 20% while for intermediate braces it is of the order of 14% to 18 %. With increase in panels from three to six, while base shear reduces by 50% and moments in columns and braces reduce almost by 20%, the staging top deflection increases excessively by 70%.

From Table 13 it is observed that, deflection at top of staging decreases up to 11% with introduction of FSJC for three to six panels.

Increase in base shear is about 11% to 13%. Bending moment in external column at footing top increases by 3% to 4% with introduction of FSJC, the effect is 4% to 5% for central columns. Maximum increase in bending moment at top of staging is up to 16% for external columns, while it is 7% to 10% for middle and central columns.

Increase in bending moment in ground brace is of the order of 15% to 18% for intermediate braces it is of the order of 10% to 18%.With increase in panels from three to six, base shear reduces by 40% and moments in columns and braces reduce almost by 35%.Correspondingly the staging top deflection increases almost by 50% to 60%.

CONCLUSIONS

The above study indicates, following observations.

1. It can be seen that the expressions reported for lateral and torsional stiffness's give results in good agreement with FEA without drudgery of 3D analysis which is practically impossible for hand calculations.

2. Lateral period in tank empty and tank full case, is less than 10% to 15% when FSJC is considered.
3. The torsional stiffness increases 10% to 15% when FSJC is considered. However, the ratio of torsional to lateral period τ is more or less not affected by application of FSJC. For four, nine and twelve columns staging the ratio 'τ' is within the critical range of $0.7<\tau<1.25$ for tank empty case. This case needs more attention as it is not always possible that tank is full with water. For tank full case generally 'τ' is outside the critical range except when number of panels are more than six.
4. Buckling factor increases with application of FSJC to the tune of 12% to 25%.
5. Deflection at tank top decreases 8% to 10% when FSJC is applied to whole staging.
6. Base shear another important dynamic parameter increases by 8% to 12% with inclusion of FSJC.
7. Bending moment in external as well as internal columns at footing top and staging top increases by 5% to10% for all types of staging and number of panels.
8. Brace moments are increased by about 10% to 20% in ground as well as intermediate braces for all the 3 types of staging with all number of panels.
9. Standard codes on staging of water towers shall prescribe that analysis of staging shall be done considering FSJC. In the absence of this analysis the values of various forces in column as well as braces shall be increased by at least 10% than those obtained with normal analysis.

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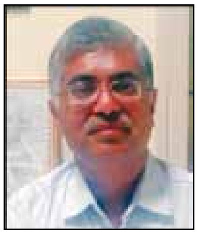

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